# M.A.M SCHOOL OF ENGINEERING 




Head of the Department Aeronautical Engineering M.A.M.School of Engineerint Siruganur,Trichy - 621105.

## Contents

- The primary purpose of this reader is to describe the use of analysis equations and methodologies of structures for design purposes. It is common these days that we hear from industry that the students graduating with engineering degrees do not know how to design whether it is structures, or mechanical systems, or systems from other engineering disciplines. We therefore put the emphasis in this course on design.
- Anybody who has some understanding of the design process, however, realises that without a thorough understanding of the use of analysis methods it will not be possible to design at least a reliable system. It is, therefore, important to establish a sound and firm analysis foundation before one can start the design practice.
- The approach used in this reader, however, is different from the traditional one in which analysis and designs are taught in different portions of the course. Instead we will use an approach in which small portions (sections) of topics from analysis are first introduced immediately followed by their design implementation.
- A more detailed description of the outline and the contents of the reader is provided in the following. The reader is divided into two primary sections because of a very important concept, called statical determinacy, that has a very strong influence on the way structures are designed. It is of course too early to completely describe the impact of statical indeterminacy on design.
- It will be sufficient to state at this point that statical determinacy simplifies the design process of a structure made of multiple components by making it possible to design individual components independent from one another. Structural indeterminacy on the other hand causes the internal load distribution in a given structural system to be dependent on the dimensional and material properties of the individual component that are typically being designed. T
- hat is, as the design of the individual components change the loads acting on those components for which they are being designed also change. Hence, individ1 2 ual components cannot be designed independently as the design changes in these components and the other components around them alter the loads that they are being designed for.
- The resulting process is an iterative one requiring design of all the individual components to be repeated again and again until the internal load redistribution stabilises, reaching an equilibrium state with the prescribed external loading.



## Contents

- Composite materials are widely used in the Aircraft Industry and have allowed engineers to overcome obstacles that have been met when using the materials individually.
- The constituent materials retain their identities in the composites and do not dissolve or otherwise merge completely into each other. Together, the materials create a 'hybrid' material that has improved structural properties.
- The development of light-weight, high-temperature resistant composite materials will allow the next generation of high-performance, economical aircraft designs to materialize. Usage of such materials will reduce fuel consumption, improve efficiency and reduce direct operating costs of aircrafts.
- Composite materials can be formed into various shapes and, if desired, the fibres can be wound tightly to increase strength. A useful feature of composites is that they can be layered, with the fibres in each layer running in a different direction.
- This allows an engineer to design structures with unique properties. For example, a structure can be designed so that it will bend in one direction, but not another.
- n a basic composite, one material acts as a supporting matrix, while another material builds on this base scaffolding and reinforces the entire material. Formation of the material can be an expensive and complex process.
- In essence, a base material matrix is laid out in a mould under high temperature and pressure. An epoxy or resin is then poured over the base material, creating a strong material when the composite material is cooled. The composite can also be produced by embedding fibres of a secondary material into the base matrix.
- Composites have good tensile strength and resistance to compression, making them suitable for use in aircraft part manufacture. The tensile strength of the material comes from its fibrous nature. When a tensile force is applied, the fibres within the composite line up with the direction of the applied force, giving its tensile strength.
- The good resistance to compression can be attributed to the adhesive and stiffness properties of the base matrix system. It is the role of the resin to maintain the fibres as straight columns and to prevent them from buckling.
M.A.M SCHOOL OF ENGINEERING

Siruganur, Tiruchirappalli - 621105.

## Teacher Teach Teachers

Report<br>The Session was initiated by Mr. G. Rajesh Kumar, AP/CSE, where he started describing about "Quantum Computing"

The agenda includes the seminar on

- Introduction of Quantum Computing
- The need for speed
- Classical vs Quantum bits
- Quantum Computing Power
- Practical Quantum Computer Applications
- Quantum Computing History
- Quantum Computing Problems

Then the session came to an end with the hand on programming with Quantum Computing.


HOD

## Teacher Teach Teachers

Date: 26-10-2019

## Speaker:

Mr.G.Rajesh Kumar, M.E.,
Assistant Professor/ CSE

## Staff Attended :

1. Mr. T.Ashok
2. Ms. S.Murugavalli
3. Mrs. D.Sumathi
4. Mrs.V.Vidhya
5. Mr. S.Nayagan
6. Mr. K.Sathish Kumar

Topic : Quantum Computing.

Venue : AB105 CSE Department

Enclosure : Report, PPT.


PRINCIPAL

## Quantum Computing

The Next Generation of Computing Devices?

by Heiko Frost, Seth Herve and Daniel Matthews

## What is a Quantum Computer?

## >Quantum Computer

> A computer that uses quantum mechanical phenomena to perform operations on data through devices such as superposition and entanglement.
$>$ Classical Computer (Binary)
> A computer that uses voltages flowing through circuits and gates, which can be calculated entirely by classical mechanics.

## Classical vs Quantum Bits

## Classical Bit

- 2 Basic states - off or on: 0, 1
> Mutually exclusive
Quantum Bit (Qubit)
- 2 Basic states - ket 0 , ket 1: $|0\rangle,|1\rangle$
> Superposition of both states (not continuous in nature)
> Quantum entanglement > 2 or more objects must be described in reference to one another
> Entanglement is a non-local property that allows a set of qubits to express
superpositions of different
binary strings (01010 and
11111, for example) simultaneously


## Pure Quibit State:

$\Psi=a|0\rangle+b|1\rangle$
where $a, b \in £$
s.t. $1=\sqrt{|a|^{2}+|b|^{2}}$
$\therefore 8$ Possible States per Qubit

- Harnesses the power of atoms and molecules to perform memory and processing tasks
- Parallel Processing - millions of operations at a time
- 30-qubit quantum computer equals the processing power of conventional computer that running at 10 teraflops (trillions of floating-point operations per second).


## Quantum Computing Power

> Integer Factorization
-Impossible for digital computers to factor large numbers which are the products of two primes of nearly equal size

- Quantum Computer with $2 n$ qubits can factor numbers with lengths of $n$ bits (binary)
>Quantum Database Search
> Example: To search the entire Library of Congress for one's name given an unsorted database...
> Classical Computer - 100 years
$>$ Quantum Computer - $1 / 2$ second


## Quantum Computing History

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1977 Aliexencer Howvo Pubiuros Pa
```


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1995 - David Deutsch of the University of Oxford, describes the first universal quantum computer



1986-Lov Griver, at Bel Lata, menens guartum datacose saerch ajogoribm
1997 - David Cory, A.F. Fahmy, Timothy Havel, Neil Gershenteld and isaace Chuang publish the first paper

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single, smair morecut
float in a cup of wat
mes. Fivat

2000 - Pioz workne 5 -quat NikR compiahar commin
2001 - Firat working 7 -qubit NMR computer demonstrated at IBA's Almaden Research
First execution of Shor's algorithm.
molecules, each containing 7 atoms.

## Practical Quantum Computer Applications

## > Quantum Mechanics Simulations

$>$ physics, chemistry, materials science,
nanotechnology, biology and medicine.
$>$ Computer can compute millions of variables at once.

- Computer can compute millions of variables at one limited today by the slow speed of quantum
-All are mechanical simulations.
$>$ Cryptoanalysis
> Capable of cracking extremely complicated codes $>$ RSA encryption



## Candidates for Quantum Computers

```
Superconductor-based quantum computers
        (including SQUID-based quantum computers)
r Ion trap-based quantum computers
- "Nuclear magnetic resonance on molecules in solution"-based
> "Quantum dot on surface"-based
> "Laser acting on floating ions (in vacuum)"-based (Ion trapping)
> "Cavity quantum electrodynamics" (CQED)-based
- Molecular magnet-based
> Fullerene-based ESR quantum computer
> Solid state NMR Kane quantum computer
```


## antum Computing Problems

Current technology
$>\approx 40$ Qubit operating machine needed to rival current classical equivalents.
$>$ Errors
> Decoherence - the tendency of a quantum computer to decay from a given quantum state into an incoherent state as it interacts with the environment.

- Interactions are unavoidable and induce breakdown of information stored in the quantum computer resulting in computation errors.
> Error rates are typically proportional to the ratio of operating time to decoherence time
- operations must be completed much quicker than the decoherence time.


## Research References

> hitp://uww.qubit.org

- nttp://www.cs.caltech.edu/-westside/quantum-intro. html
> http://computer.howstuffworks.com/quantum-computer1. htm
> http://en wikipedia.ora/wiki/Quantum computers
> http://mww.carolla.com/quantum/QuantumComputers.htm



## M.A.M SCHOOL OF ENGINEERING

 Siruganur, Tiruchirappalli - 621105.
## Teacher Teach Teachers

Date: 11-01-2020

## Speaker: <br> Mr.K.Sathish Kumar, M.E., <br> Assistant Professor/ CSE

Staff Attended :

1. Ms. S.Murugavalli
2. Mr.G.Rajesh Kumar
3. Mrs.V.Vidhya
4. Mrs.P.Sivamalar
5. Mr. K.Sathish Kumar
6. Mrs. D.Sumathi

Topic: Graphics \& Multimedia.

Venue : Peter Norton Lab

Enclosure : Report, PPT.

M.A.M SCHOOL OF ENGINEERING

Siruganur, Tiruchirappalli - 621105.

## Teacher Teach Teachers

## Report

The Session was initiated by Mr. K.Sathish Kumar, M.E., AP/CSE, where he started describing about "Graphics \& Multimedia"

The agenda includes the seminar on

- Introduction of Graphics \& Multimedia
- Graphical Representation
- Features of Graphics.
- Types of Media
- Multimedia Applications
- Advantages of Multimedia
- Difference between 2D \& 3D Graphics.

Then the session came to an end with the hand on creative designing with Graphics \& Multimedia software tools.



## WHAT IS MULTIMEDIA?

Multimedia - using more than one media:

- Text
- Graphics
- Animation
- Sound
- Video


## WHAT IS MULTIMEDIA?

- In a generic sense, multimedia is simply the use of more than one media element. Hence, Web-based multimedia is defined as an online, interactive experience that incorporates two or more media elements including text, graphics, sound, animation and video. A fundamental feature of most Webbased multimedia is interactivity, which gives user some control over the content.


## Graphical Representation

- Today, this integration is accomplished by digitizing different media elements and then manipulating them with computer software
- Digitized - Media elements have been captured in a code that the computer can understand.



## IMPORTANCE OF MULTIMEDIA

- "Tell me and I. will forget; show me and I may remember; involve me and I will understand" (Chinese proverb)
- Each person learns differently and each person is inspired by something different. The use of multimedia allows developers to tap into these differences.


## IMPORTANCE OF MULTIMEDIA

- In fact, research shows that people remember only $20 \%$ of what they see, $30 \%$ of what they hear. When they see and hear it, they remember $50 \%$, if we include some interaction; they will remember $80 \%$ of it


## ADVANTAGES OF MULTIMEDIA

WHERE DO WE USE MULTIMEDIA?

- Addresses multiple learning styles
- Provides an excellent way to convey content
- Uses a variety of media elements to reinforce one idea
- Activates multiple senses creating rich experiences
- Gives life to flat information
- Enhances user enjoyment
- Improves retention
- Enables users to control Web experience
- Multimedia in Business

Business application that are multimedia based include Business application that are multimedia based include
presentations, training, marketing, advertising, product presentations, training, marketing, advertising, pros,
demos, databases, catalogues, and networked communications. Multimedia is getting much utilization in training programs.

- Multimedia in School

Schools are perhaps the most ideal target for multimedia. Its rich set of media is potential for delivering effective teaching. Multimedia equipped education lets the students. Learn at their own pace and at their own time. It is ideal in distance education and open learning systems wherein students need not to be physically present in class. Students can learn while having fun.


- Multimedia at Home

From cooking to gardening, home design to repair, indeed. multimedia has made itseif useful at home. It enables you to convert your video to digital format, store your pictures in a compact disc, and many more. Today, multimedia is also reach out homes via interactive TV (iTV).

- Multimedia in Public Places

Multimedia is present in standalone terminals, or kiosks, in Multimedia is present in standals, train stations, museums,
airport terminals, hotels, mall, airport terminals, hotels, mall, train stations, museums,
grocery stores, and more. about a particular place. Interactive kiosks enables you to make a transaction without talking to a sales agent.

## WHERE DO WE USE MULTIMEDIA?

## - Multimedia in the Internet

Multimedia was introduced in the Internet with the Multimedia was introduced in the Internet wit
advent of the WWW. In fact, the Web is the advent of the WWW. In fact, the Web is the
multimedia part of the internet. In the early stages multimedia part of the internet. In the early stages of the internet, you can view information in plain text. The Web enables multimedia to be delivere online. Playing live Internet games with multip
players around the world has caught much players around the world has caught much
attention. Some e-learning systems use multimedia on the internet as a method to deliver learning materials to students anywhere.

## WHERE DO WE USE MULTIMEDIA?

- Multimedia in Mobile Devices

Mobile devices such as personal digital assistants (PDAs or handheld computers), smartphones, and mobile devices are not exceptions to multimedia. MMS (Multimedia Messages Services) is a store-and-forward method of transmitting graphics, video clips, sound files, and short text messages over wireless network using the WAP. It also supports email addressing, so the device can send-emails directly to communication between mobile phones.

## WEB-BASED MULTIMEDIA CATEGORIES

- Electronic Commerce (E-Commerce) Involves using web to serve clients and customers and is one way to provide solutions for companies that wish to sell products or services online. that wish to sell products or services onerine. marketing.
- Web-Based Training and Distance Learning

The Web offers many options for delivering and receiving education over the distance. Web-based training is an instruction delivered over the Internet using a web browser.


- Research and References

Today newspaper, newsletters, magazines, books, encyclopaedias and other reference materials are being offered online via Web. In many cases, they represent "Electronic" versions of existing research and reference materials. An increasing number of self-help and how guides are being offered as interactive muitimedia referencing, Expanded search capabilities, multisensory experiences.

- Entertainment and Games

They are the examples of some of the most popular and most varied interactive multimedia sites available.

Difference between 2D \& 3D
Graphics.
2D
Vector Graphics - It works with lines \& cures, When we zoom or scale the object it will redraw the shape

3d
$X, Y \& Z$ axis real time object viewing.

## Management-Related Positions

- Executive Producer - Move a project into an through production
- Project Manager - forming a project, moving it into production and overseeing its creation


## Production-Related Positions

- Audio Specialist - Music scores, sound effects, voice overs, vocals and transitional sounds, recording, editing and selecting voices, sounds and music
- Computer Programmer - Creates the underlying code that makes the website interactive and responsive to user's actions
- Video Specialist - Manages the process of capturing and editing original video


## Production-Related Positions

- Web Designer - Develops or refines a design process and efficiently creates a cohesive and wellplanned website from the front end
- Web Developer - Ensures the communication between the front end of the website and its back end is working
- Web Master - Making sure the web page is technically correct and functional on the Web Server


## Art-Related Positions

- Animation Specialist - Creates 2D/3D animation by taking a sequence of static images and displaying them in rapid succession on the computer screen
- Art Director - Coordinate the creation of the artwork for the project
- Graphic Artist/Designer - Creating and designing all of the graphic images for a project


## Art-Related Positions

- Interface Designer - Responsible for the look of the website interface and navigation methods


## Content-Related Positions

- Content Specialist - Providing authenticity and accuracy of information on the website
- Instructional Specialist - Expert in designing instructional projects
- Photographer - Shoots and captures appropriate, compelling and high quality photos
- Writers/Editors - Technical writers/scriptwriters, creative writers or
- Videographer - Shoots and captures journalist involved in the project


## Thank You.

## M.A.M SCHOOL OF ENGINEERING

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# Department of Electrical and Electronics Engineering <br> Academic year: 2019-2020-ODD 

## TEACHER TEACH TEACHERS

## Speaker. Mr. PURUSHOTHAMAN

HOD / ERE
Department of EEE
Staff Attended:

1. Mr.A.Senthamarai kennan Asso.Prof/EEE
2. Mr. M. Ranjith kumar $A P / E E E$
3. Mr. G.Purushothaman HOD/EEE
4. Ms.K.Vinothini AP /EEE
5. Mr. Ismail Gan AP/EEE

Topic:

## Electrical Measurement and Instrumentation

## Venue:

Circuits Lab

Report encl:
$21^{\text {st }}$ July $2019 \& 1.30$ PM to 2.30 PM





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ALIYGENIT 9
which has a reading range of $-100^{\circ} \mathrm{C}$ to $100^{\circ} \mathrm{C}$ So
Span is $200^{\circ} \mathrm{C}$.
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given reading would be 71 kg . This would indicate that there
is a constant bias of 1 kg to be corrected. Therefore, if A with a weight of 70 kg weighs himseeff, the


 5. BIAS


$\Delta \theta_{0}$ :change in output; © 0 : change in inpout
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STATIC CHARACTERISTICS




## STATIC CHARACTERISTICS

DYNAMIC CHARACTERISTICS Romp Input
The signal changes linearly.
The output signal for ramp input is 'ramp
response'. $\square-2+$


## 

 such as 'step input', 'ramp input' and 'sine-wave input'.
-

Explains the behaviour system of
Instruments system when the input signal is
changed. Explains the behaviour system of

## Stons3 10 sodA1


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## M.A.M SCHOOL OF ENGINEERING

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Trichy - Chennai Trunk Road, Siruganur. Trich! - 6,21116

## Department of Electrical and Electronics Engineering

Academic year: 2019-2(120 1 1:1 1in

## TEACHER TEACH TEACHERS

Speaker: Mr. MEENAKSHI M.
Assistant Professor
Department of EEE

## Staff Attended:

1. Mr. G.Purushothaman HOD/EEE
2. Mr.A.Senthamarai kannan Asso.Prof/EIEI:
3. Mr. Ranjith Kumar AP/EEE
4. Ms. Vinothini AP/EEE
5. Mr. Ismail Gani AP/EEE

Topic:

> "Design of Solar Panel Standalone Home Load"

## Venue:

Circuits Lab

Report encl:
$25^{\text {th }}$ February 2020 \& 1.30 P.M to 2.30 P.M



By
M.Meenakshi/AP/EEE
M.A.M School of Engineering

Major System Components \&It's Working

- Solar PV panel(solar PV module)

PV module converts sunlight into DC electricity.

- Solar charge controller Charge controller regulates the voltage and current coming from the PV panels going to the battery.
- Inverter Inverter converts DC output of PV panels into pure $A C$ power to the $A C$ applications.
- Battery bank Battery stores the energy for supplying to electrical appliences when there is a demand

Determination of Total Load
Determine the power consumption demands, calculate total watt-hour per day used by the appliences

| 5ino |  | POWER(wats) | MOUR/DAY | Buchy |
| :---: | :---: | :---: | :---: | :---: |
| 1 | PORTIGO LIGHT | 12 | 2 | 24 |
| 2 | HALL LIGHT | 40 | 2 | 80 |
| 3 | HALL FAN | -90 | 2 | 180 |
| 4 | KITCHEN LIGHT | 40 | 1 | 40 |
| 5 | ROOM UGHT | (r8\% 12 | 1 | 12 |
| 6 | ROOM FAN | 90 | 2 | 180 |
| 5 | 230 | 284 28 | - | -516 |

Total energy required per(watt-hour) $=516 \mathrm{~Wh}$ Total Load $=284$ Watts

Solar Panel With Home Load


Design Procedure For Solar PV system
Step1: Determine the total load of the homm/haty
Step2:Calculate the number of paneis
Step3: Determine the size of the batterv
Step4: Determine the rating of the chatre controller
Step5: Determine the rating of the inverter.

Solar PV system Panel design

Total energy required per da, wh:
Formula $=$



# Solar PV 5ystem Panel design <br> Formula $=\frac{\text { Total energy required per day(Wh) }}{\text { panel power gen factor(0.45) 'power gen hours }}$ <br> 516 <br> $=$ <br> <br> $0.45^{*} 6$ hours <br> <br> $0.45^{*} 6$ hours <br> $=$ rounded of 250 Wp (chose $125 \mathrm{Wp}^{\circ} 2$ nos) panel specification on board( $125 \mathrm{Wp}, 12 \mathrm{v}$ ) 

## Design Of Charge Controller

Charge Control Rating $=$ Panel Isc $\times$ Number OP Panel $\times 13(30 \%$ ) $\rightarrow$
Charge Controd Rating $=7.5 \times 2 \times 1.3=19.5$ Amps

## Charge Control: $=2 \mathrm{Amps} \times 12 \mathrm{~V}$

```
                        Battery design
Battery Capactly (Ah)=
```


Batery Eficuency $(0.85) \times$ Batery $000(06) \times$ E Brery valdge: $12 \times 24$
Axter Cocpaty A


## Inverter design Procedure

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" 1.1 "." -




## Design of Inverter



## Ratings of the solar PV system



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    Ratest twat!,
```



```
    1125 Wra = I -V bririts
```



```
    Charge Control =20 Amps }\times12\textrm{V
    Invatter = 500 VA
```

With the help of these procedure we can desifn a soldi 9 , whtere for any kind of stand Alone load

## M.A.M SCHOOL OF ENGINEERING

(Aceredited be N. Til::)



## Department of Electrical and Electronics Enцinecring

Academic year: 2019-2020 - ISIIN

## TEACHERTEACH TEACHERS

Speaker: Ms. DHANALAKSHMI D,
Assistant Professor
Department of EEE:
Staff Attended:

1. Mr. G.Purushothaman II()| /IE! $1:$
2. Mr.A.Senthamarai kannan Asso, Prof/lilit:
3. Mr, Ranjith Kumar AP/EISI:
4. Mrs. Meenakshi $\Lambda \mathrm{P} /$ EEE
5. Ms. Vinothini AP/EISE
6. Mr. Ismail Gani $\lambda P / I \leq I S!$

## Topic:

> "Artificial Intelligence"

## Venue:

## Circuits Lab)

Report encl:
$4^{\text {th }}$ February 2020 \& 1.30 P.M to $2.30 \mathrm{P}, \mathrm{N}$


## Artificial Intelligence

## What is Artificial Intelligence? <br> 



Robots at Artemis


Artificial Intelligence Tests
Turiag Teut

Artificial Intelligence \S. Robot


Artificial Intelligence Tert
$\qquad$
$\qquad$

Artificial Intelligence Tests

Fina les inmiver an interpeter, a male, amela lentic
Female peetends to be male
Inteppeter tries to fipure out who is whe
Scound teat is similar to turing tees
Comprores tovh leets

Artificial Intelligence Teかっ
Iminu Iow


## Examples



Problems


## AI Controversies

## Potatidel jub talouver



## M.A.M SCHOOL OF ENGINEERING

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Trichy - Chennai Trunk Roacl, Siruganur, Trichı

## TN

## Department of Electrical and Electronics Engineering <br> Academic year: 2(01)-2(12(1)-()|)1)

## TEACHER TEACH TEACHERS

## Speaker: Mr. RANJITH KUMAR M.

Assistant Professor.
Department of EEE

## Staff Attended:

1. Mr.A.Senthamarai kannan Asso.Prof/EIEI:
2. Mr. G.Purushothaman HOD/EEE:
3. Ms.K.Vinothini AP/EEE
4. Ms.Dhanalakshmi AP/EEE
5. Mr. Ismail Gani AP/EEE

## Topic:

## "Power System"

## Venue:

Circuits Lab

Report encl:
$19^{\text {dh }}$ August $2019 \& 1.30$ P.M to 2.30 P.MI


MANAGEMENT AND OPTIMIZATION OF SOLAR POWER CONVERSION TO SUPPLEMENT TERRESTRIAL POWER SYSTEMS.

BY

Mr. RANJITH KUMAR AP / EEE MA.M SCHOOL OF ENGIEERING

ENVIRONMENTAL ASPECTS OF ENERGY

- TRADE-OFF BETWEEN ENERGY AND ENVIRONMENT
- ECOLOGICAL UNBALANCE
- GLOBAL WARMING
- RADIATION HAZARDS


## CONVENTIONAL RESOURCES

- FOSSIL FUELS
- HYDRO RESOURCES
- NUCLEAR RESOURCES USING FISSION


## NON-CONVENTIONAL RESOURCES

- SOLAR ENERGY
- WIND ENERGY
- BIOMASS ENERGY
- OCEAN WAVE ENERGY
- OCEAN THERMAL ENERGY CONVERSION
- GEOTHERMAL ENERGY
- OCEAN TIDAL ENERGY

AND

- NUCLEAR FUSION

LIMITATIONS OF SOLAR ELECTRIC POWER GENERATION INSTALLATIONS ON EARTH

- Effects of day/night cycles
- Shadowing due to clouds, fog, snow, precipitation etc.
- Weather effects
- Reduced solar-radiation intensity
- Overall variable and discontinuous power output

LIMITATIONS OF SOLAR ELECTRIC POWER GENERATION INSTALLATIONS ON EARTH

HENCE:

SOLAR POWER SATELLITE CONCEIVED

## SALIENT ADVANTAGES OF SPS

- More intense (about eight times on average) solar radiation available
- Unaffected by weather, clouds etc.
- SPS illuminated almost all the time (except eclipse periods). Hence expensive storage not required
- Lack of gravity simplifies structure
- Waste heat re-radiated back into space, instead of warming the biosphere.


## SOLAR TO ELECTRIC CONVERSION

## Contd....

## POINTS FOR CHOICE:

- Energy conversion efficiency
- Cost effectiveness
- Material and system transportation convenience
- Technology status
- Specific feasibility problems

SOLAR TO ELECTRIC CONVER

- THERMAL ELECTRIC CONVERSIC.
- SOLAR DYNAMIC CONVERSION
- DIRECT CONVERSION THROUGH PHOTOVOLTAICS

MAIN PARAMETERS FOR CONSIDERATION OF PHOTOVOLTAIC POWER GENERATION ON SFS

- Energy conversion efficiency
- Life-expectancy
- Tolerance to space-radiation environment
- Power-production capacity per unit araa
- Production cost including marera processing cost

MAIN PARAMETERS FOR CONSIDERATION OF PHOTOVOLTAIC POWER GENERATION ON SPS

Contd.....

- Amenability to mass production
- Consideration for optimized mass
- Overall bulk and portability



## BRIEF HISTORICAL MILESTONES

－1899－1900 NIKOLA TESLA Proposed use of radio waves power transmission
－1930＇s Use of microwaves proposed for power transmission
－ 1945 Clarke putforth the concept of geo－stationary satellite in Science－fiction
－ 1962 Satellite communication begins with Telstar I first rectenna build

BRIEF HISTORICAL MILE－STONES

## （Contal．．）

－ $197584 \%$ efficient microwave to DC conversion demonstrated
－ 1983 US Patent for a system for power transmission from SPS \＆direct conversion to $60 \mathrm{~Hz}, 3$－Phase
－1999－2000 SPS Exploratory Concept examined by NASA


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        helicopter using 2&5GHz
.1954 IEEE Corderence sNE'ビご
    session anMじつまミッチニこッに
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    Satelines
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    earh
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## BRIEF HISTORICAL MLLE－STONES

－2001－2002 Technology Maturaten program for SPS pursuev dy ivizi
－ 2004 A Report on possible design ré SPS prepared oy NASA
 considering such projects lak announced plans to nave ts＂－ジミー・ operation by $20: 0$

## METHODOLOGY OF TRANSMISSION AND UTILIZATION

－DIRECT TRANSMISSION MICROWAVE PO：RER SMALLER ANTENNA FEEDING TO UTILITY SYSTEM EQUIFIEN
－TRANSMISSION TO LARIGE CENTRA EARTH STATION \＆FEEDING to TERRESTRIAL POWER SYS：EN （POWER GRID）

## MAIN SUBSYSTEMS OF SOLAR POWER SATELLITE

- SOLAR POWER COLLECTOR AND SUBTRACKER SUB-SYSTEM
- POWER CONVERSION SUB-SYSTEM
- POWER TRANSMISSION SUB-SYSTEM
- TELEMETRY, TRACKING \& COMMAND SUBSYSTEM
- ANTENNA SUB-SYSTEM
- PROPULSION \& ATTITUDE STABILIZATION SUB-SYSTEM


## SOLAR POWER SATTELLITE LOCATION OPTIONS

- GEO-SYNCHRONUS-STATIONARY ORBIT ( 3600 KM FROM EARCH)
- MEDIUM EARTH ORBIT (MEO AROUN: 10000 KM)
- LOW EARCH ORBIT (LEO AROINT 800-1000 KM)
- HIGH ALTITUDE PLAIFORM (HAK. LESS THAN 100 KM)



## M.A.M SCHOOL OF ENGINEERING

(Accredited bo N.IV)

Trichy - Chemai Trunk Road, Sirmganme, 'Trichy 6..11 165

## Department of Electrical and Electronics Enginecring

Academic year: 2019-2020 - ODD

## TEACHER TEACH TEACHERS

## Speaker: Mr. ISMAIL GANI M.

Assistant Professor
Department of EEE
Staff Attended:

1. Mr. G.Purushothaman HOD/ELEL:
2. Ms.K.Vinothini AP/EEE
3. Ms.Dhanalakshmi AP/EEE
4. Mr.A.Senthamarai kannan Asso.Prof/IIII:
5. Mr. Ranjith Kumar AP/EEE.

## Topic:

> "Synchronous Motor"

## Venue:

Circuits Lab

Report encl:
$4^{\text {th }}$ October 2019 \& 1.30 P.M to 2.30 P.M


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## M.A.M SCHOOL OF ENGINEERING,

(Accredited by NAAC)

Trichy - Chennai Trunk Road, Siruganur, Trich 621115

## Department of Electrical and Electronics Enginecring <br> Academic year: 2019-2()20) - $1 \vdots 11 \vdots$

## TEACHER TEACH TEACHERS

## Speaker: Ms. VINOTHINI K.

Assistant Professor
Department of EEE
Staff Attended:

1. Mr. G.Purushothaman HOD/EEI:
2. Ms.Dhanalakshmi AP/EEE
3. Mr.A.Senthamarai kannan Asso.Prof/EEE
4. Mr. Ranjith Kumar AP/EEE:
5. Mrs. Meenakshi AP/EEE
6. Mr. Ismail Gani AP/EEEI:

Topic:
"Solar Tree"

Venue:
Circuits Lab

Report encl:
$10^{\text {dh }}$ January 2020 \& 1.30 P.M to 2.30 P.M


## INTRODUCTION <br> - solar energy is the best option. - Solar tree sounds like the perfect solution for our future energy needs. - A solar tree is an artificial tree with photo-voltaic cells arranged in Fibonacciseries manner. - Uses multiple no. of solar panels which forms the shape of a tree.

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## WHAT IS SOLAR TREE

| A solar tree is a decorative means of protucing solur energe and also elestricity. It uses multiple no of solar panels which firms the shape of a tree The panels are arranged in a tree fishion in a tall towerpole |
| :---: |
| TreE stands for |
| T= TREE GENERATING |
| R -RENEM CBIE |
| $\mathrm{t}=1$ NER() $\mathrm{I}_{\text {ad }}$ |
| E) lectrictr |
|  the tee whin mivur enty |

## max

## - Introduction. - History. What is SOLAR TREE? - What is SOLAR TRe - Components of Solar Tree Why we called it as solar tree <br> Why we called <br> Need of Solar Tree Why it is better than traditional system <br> Solar Tree in India <br> Application <br> - Adrantages. - Disidvantages. - Future of Solar Tree - Conclusion <br> 





## HISTORY



COMPONFNTSOFSOI AR TRFE



WHY IT IS BETTER THAN A

inverter． alternating 4ion suomojp oulu 14 ！！ electricity by converting photons of ？

it 50.108

##  <br> （6）

AdVantages of Solar Panel


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－Decreased Electrical Bill
－Ecölogically Friendly

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## GIBL YVTOS AO dA3N

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CONCLUSION
o To fulfils the increasing energy demand the people.

- Saving of land this project is very successful one
o This can provide electricity without any power cut
problem.


## M.A.M SCHOOL OF ENGINEERING;

(Accreditedlen N.IM:)
 Trichy - Chennai Trunk Road. Sirmpanm. Trulos 6.21 min

## Department of Electrical and Electronics Enginecring <br> Academic year: 20192020 (川)!

## TEACHERTEACHTEACHERS

Speaker: Mr. SENTHAMARAI KANNAN A.
Associate Professor
Department of EEI:
Staff Attended:

1. Mr. G.Purushothaman HOD/EEE
2. Ms.K.Vinothini AP/EEE
3. Ms.Dhanalakshmi $\mathrm{AP} / E E I:$
4. Mr. Ismail Gani $A P / I: I: I:$
5. Mr. Ranjith Kumar AP/ELLL

## Topic:

## "Electrical Machines"

## Venue:

## Circuits Lab

## Report encl:

$13^{\text {di }}$ September 2019 \& 1.30 P.M1 102.30 P... 1



$3 \pi$ terminals secondiv, the method is nit verveflicient sinc
resistaice and operation at high slip entals dissipation. terminals are available outside. For cige rotor machmes, there are in tutor




 shown in fig. 18. These curves show that the slip at maximum torques ${ }^{-}$ see that the torque depends on the square of the applied voltage. The
variation of speed torque curves with respect to the applied voltage is
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 line at some point. This point is marked as $f$ in the given curve.

 power to the induction generator as well as load.
$>$ The function of the capacitor bank is to provide the laggingreactive is connected across its stator terminals
why it is called self excited? It is because it uses capacitor bank which



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(Approved by AICTE, New Delhi Affilated roy. Inna L inversme, (hemm..n)
Trichy - Chennai Trunk Road, Siruganur, Trichy-6,21105

## Department of Electrical and Electronics Engineering <br> Academic year: 2019-2120 - 1: \1:

## TEACHER TEACH TEACHERS

Speaker: Mr. PURUSHOTHAMAN
HOD / EEE
Department of EEE
Staff Attended:

1. Mr.A.Senthamarai kannan Asso.Prof/EIE:
2. Mr. M.Ranjith kumar AP/EEE
3. Mr. G.Purushothaman HOD/EEE
4. Ms.K.Vinothini AP/II:I
5. Mrs. Meenakshi M AP/I:I:I:
6. Mr. Ismail Gani AP/EEE

Topic:
"Three Phase Transformers"

Venue:
Circuits Lab

Report encl:




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## REPORT

The session was initiated by Mr. K.Karthikeyan Assistant Professor/ECE. He explained about ARM Processor and its Applications and discuss about the following topics

- ARM architecture
- Instruction set
- LPC 214X-family
- The timer unit
- Pulse width modulation unit
- Block diagram of ARM 9

The session comes to an end with the explaining the overview of ARM Processor and its Application.


## MAM SCHOOL OF ENGINEERING <br> Siruganur, Tiruchirappalli-621 105. Academic year (2019-2020) Even semester

Department of Electronics \& Communication Engineering

## Teacher Teach Teacher (TTT)

Date:16-03-2020
Speaker: Mr.K.Karthikeyan
Assistant professor-Electronics and Communication Engineering Staff attended:

1. Mrs.K.Umarani
2. Mr.G.Sathesh kumar
3. Mr.A.Karthick kumar
4.Mr.Arumugasamy
5.Mr.K.Saravanan
4. Mrs.Berbeth mary
7.Mrs.A.Subha pradha

## Topic:

ARM Processor

## Venue:

Smart class
Date \& Time:
$16^{\text {din}}$-march-2020\&1.30 pm -2.30 pm
**enclosure: Report

## REPORT

The session was initiated by Mr.A. hathich humar. Assistant Professor/ECE. he explained about Embedded sy sem dewign prowe and disenss about the following topics

- Design model
- Design methodologies
- Design flows
- Requirement analysis
- Quality assurance techniques
- Designing with computing platform

The session comes to an end with the explaining the orentell of tmbedded system design process and its Application.


## MAM SCHOOL OF ENGINEERING

## Department of Electronics \& Communication Engineering

## Teacher Teach Teacher (TTT)

Date: 09-03-2020
Speaker: Mr.A.Karthick kumar
Assistant professor-Electronics and Communication Engineering

## Staff attended:

1. Mrs.K.Umarani
2. Mr.G.Sathesh kumar
3. Mr.K.Karthikeyan
4. Mrs.Berbeth mary
5.Mrs.A.Subha pradha
6.Mr.Arumugasamy
7.Mr.K.Saravanan

Topic:
Embedded system design process

## Venue:

Smart class

Date \& Time:
$9^{\text {th }}$ march -2020\&1.30 pm to 2.30 pm
**enclosure: Report


The Acorn Business Computer ( ABC ) plan required that a number of second processors be made to work with the BBC Micro platform, but processors such as the Motorola 68000 and National Semiconductor 32016 were considered unsuitable, and the 6502 was not powerful enough for a graphics-based user interface. ${ }^{[19]}$

According to Sophie Wilson, all the processors tested at that time performed about the same, with about a $4 \mathrm{Mbit} /$ second bandwidth. ${ }^{[20]}$

After testing all available processors and finding them lacking, Acorn decided it needed a new architecture. Inspired by papers from the Berkeley RISC project, Acorn considered designing its own processor. ${ }^{[21]} \mathrm{A}$ visit to the Western Design Center in Phoenix, where the 6502 was being updated by what was effectively a single-person company, showed Acorn engineers Steve Furber and Sophie Wilson they did not need massive resources and state-of-theart research and development facilities. ${ }^{[22]}$

Wilson developed the instruction set, writing a simulation of the processor in BBC BASIC that ran on a BBC Micro with a 6502 second processor. ${ }^{[23 \mid[24]}$ This convinced Acorn engineers they were on the right track. Wilson approached Acorn's CEO, Hermann Hauser, and requested more resources. Hauser gave his approval and assembled a small team to implement Wilson's model in hardware. ${ }^{\text {[citation needed] }}$

## Acorn RISC Machine: ARM2[edit]

The official Acorn RISC Machine project started in October 1983. They chose VLSI Technology as the silicon partner, as they were a source of ROMs and custom chips for Acorn. Wilson and Furber led the design. They implemented it with efficiency principles similar to the 6502. ${ }^{[25]}$ A key design goal was achieving low-latency input/output (interrupt) handling like the 6502. The 6502's memory access architecture had let developers produce fast machines without costly direct memory access (DMA) hardware.

The first samples of ARM silicon worked properly when first received and tested on 26 April 1985. ${ }^{[3]}$

The first ARM application was as a second processor for the BBC Micro, where it helped in developing simulation software to finish development of the support chips (VIDC, IOC, MEMC), and sped up the CAD software used in ARM2 development. Wilson subsequently rewrote BBC BASIC in ARM assembly language. The in-depth knowledge gained from designing the instruction set enabled the code to be very dense, making ARM BBC BASIC an extremely good test for any ARM emulator. The original aim of a principally ARM-based computer was achieved in 1987 with the release of the Acorn Archimedes. ${ }^{[26]}$ In 1992, Acorn once more won the Queen's Award for Technology for the ARM.

## ARM PROCESSOR

ARM, previously Advanced RISC Machine, originally Acorn RISC Machine, is a family of reduced instruction set computing (RISC) architectures for computer processors, configured for various environments. Arm Holdings develops the architecture and licenses it to other companies, who design their own products that implement one of those architecturesincluding systems-on-chips $(\mathrm{SoC})$ and systems-on-modules ( SoM ) that incorporate memory, interfaces, radios, etc. It also designs cores that implement this instruction set and licenses these designs to a number of companies that incorporate those core designs into their own products.

Processors that have a RISC architecture typically require fewer transistors than those with a complex instruction set computing (CISC) architecture (such as the $x 86$ processors found in most personal computers), which improves cost, power consumption, and heat dissipation. These characteristics are desirable for light, portable, battery-powered devicesincluding smartphones, laptops and tablet computers, and other embedded systems ${ }^{[3] / 4][5]}$-but are also useful for servers and desktops to some degree. For supercomputers, which consume large amounts of electricity, ARM is also a power-efficient solution. ${ }^{[6]}$

Arm Holdings periodically releases updates to the architecture. Architecture versions ARMv3 to ARMv7 support 32-bit address space (pre-ARMv3 chips, made before Arm Holdings was formed, as used in the Acorn Archimedes, had 26-bit address space) and 32-bit arithmetic; most architectures have 32 -bit fixed-length instructions. The Thumb version supports a variablelength instruction set that provides both 32 - and 16 -bit instructions for improved code density. Some older cores can also provide hardware execution of Java bytecodes; and newer ones have one instruction for JavaScript. Released in 2011, the ARMv8-A architecture added support for a 64 -bit address space and 64 -bit arithmetic with its new 32 -bit fixed-length instruction set. ${ }^{[7]}$ Some recent Arm CPUs have simultaneous multithreading (SMT) with e.g. Arm Neoverse El being able to execute two threads concurrently for improved aggregate throughput performance. ARM Cortex-A65AE for automotive applications is also a multithreaded processor, and has Dual Core Lock-Step for fault-tolerant designs (supporting Automotive Safety Integrity Level D, the highest level). The Neoverse N 1 is designed for "as few as 8 cores" or "designs that scale from 64 to 128 N 1 cores within a single coherent system". ${ }^{[8]}$

With over 130 billion ARM processors produced, ${ }^{[9]|1||[1]| \mid 2]}$ as of 2019, ARM is the most widely used instruction set architecture (ISA) and the ISA produced in the largest quantity. ${ }^{[13][4][14][15][16]}$ Currently, the widely used Cortex cores, older "classic" cores, and specialized SecurCore cores variants are available for each of these to include or exclude optional capabilities. The British computer manufacturer Acorn Computers first developed the Acorn RISC Machine architecture (ARM) ${ }^{[17][18]}$ in the 1980 s to use in its personal computers. Its first ARM-based products were coprocessor modules for the 6502 B based BBC Micro series of computers. After the successful BBC Micro computer, Acorn Computers considered how to move on from the relatively simple MOS Technology 6502 processor to address business markets like the one that was soon dominated by the IBM PC. launched in 1981.
destination is found from database and displayed by the renderer. The system block diagram may be refined into two block diagrams - hardware and software.

System integration After the components are built, they are integrated. Bugs are typically found during the system integration. Good planning can help us to find the bugs quickly. By debugging a few modules at a time, simple bugs can be uncovered. By fixing the simple bugs early, more complex or obscure bugs can be uncovered. System integration is difficult because it usually uncovers problems. The debugging facilities for embedded systems are usually much more limited than the desktop systems. Careful attention is needed to insert appropriate debugging facilities during design which can help to ease system integration problems.

## Summary

In the Embedded system design process, information is first collected and refined in the Requirement step. Specification uses the relined information to describe the functions of the system which accurately reflects the customer's requirements and also serves as the contract between the customer and the designer. The functions described by the specification are implemented by the Architecture design. The architectural design describes the components we need which will include both the hardware and software components. After the components are built, they are integrated to get the final system.

## EMBEDDED SISTEM DESIGN MROCES

Design process steps: There are different siep imolwid it t mbedded system design process. These steps depend on the devign methototoys Devign methodology is important for optimizing performance, and developing woppher added design tools. It also makes communication between twans memter water. They are requirements gathering, specification fommlation, architectas devigh, buibling of components, and system integration. Figure 1. Steps in Design pherev Ite whp it the devign process can be viewed as top down view and bothom wh vell. Wep dwh vien begins with the most abstract description of the system and conclades with whetete detaik. Bottom-up view starts with components to build a system. Bothom ap dewign steps are shown in the figure as dashed-line arrows. We need bottom wip design hoans wo dh not hate perfect insight into how later stages of the design process 1 ill thm wht The matior gats of the design to be considered are : - Manufacturing cont: Pertomatos (thoth orerall speed and deadlines) and - Power consumption. The tashs which mad whepertormed at each step are the following. - We must analyze the design at cath sep to determine how we can meet the specifications. - We must then retine the divisn to add details. - We must verify the design to ensure that it still mects all system ghals, swh as cost, speed, and so on. We will discuss each step of the design process in detail. 1.1 Requirements Informal descriptions gathered from the customer are known as reyuirements. The requirements are refined into a specification to begin the designing of the sistem architecture. Requirements can be functional or non-fimetional rywhements, functional requirements need output as a function of input. Non-functional rypurememts includes performance, cost. physical size, weight, and power consumption. Neformance may be a combination of soft performance metrics such as approximate thene to perform a user-level function and hard deadlines by which a particular oporation mast be completed. Cost includes the manufacturing, nonrecurring enginecring(NRE) and other costs of designing the system. Physical size and weight are the physical aspects of the final system. These can vary greatly depending upon the application. Power consmmption can be specitied in the requirements stage in terms of battery lite.

## Architecture Design

The specification deseribes only the functions of the system. Implementation of the system is described by the Architccture. The architocture is a plan for the overall structure of the system. It will be used later to design the components. The architecture will be illustrated using block diagrams as shown blow. Example: A basic block diagram of the GPS system shows the major operations and the data flow among the blocks. Figure 3. GPS system data flow and operations This block diagram( figure 3) is an initial architecture that is not based ether on handuare or on sotwame but combination of both. This block diagram explains about GPS navisating sistem where GPS receiver gets current position and the destimation is taken from user. digital map for source to

## M.A.M. SCHOOL OF ENGINEERING DEPARTMENT OF MECHANICAL ENGINEERING

Teacher Teach Teacher (TTT) Programme

ACADEMIC YEAR 2019-20 (ODD SEMESTER)

| Sl.No. | Name of the Faculty | Syllabus | Date \& Session |
| :---: | :---: | :---: | :---: |
| 1 | Dr.P.Ranjith kumar | Computer Integrated Manufacturing | $07 / 09 / 2019$ |
| 2 | R.Ramanathan | Manufacturing Method by using Various Sheet Metals / | FN |

## M.A.M. SCHOOL OF ENGINEERING DEPARTMENT OF MECHANICAL ENGINEERING

Teachers Teach Teacher (TTT) Programme

ACADEMIC YEAR 2019-20 (EVEN SEMESTER)

| SI.No. | Name of the Faculty | Syllabus | Date \& Period |
| :---: | :---: | :---: | :---: |
| 1 | Dr.M.Panneerselvam | Design of Various Mechanical Elements. | $\begin{gathered} 03 / 01 / 2020 \\ \text { FN } \end{gathered}$ |
| 2 | Dr.K.Chandrasekaran | $\begin{aligned} & \text { Optimazation } \\ & \text { Materials }\end{aligned}$ Tools and Technique Composite | $\begin{gathered} 15 / 02 / 2020 \\ \text { FN } \end{gathered}$ |

# Introduction to Computer Integrated Manufacturing (CIM) 

1. Flexible Manufacturing System (FMS)
2. Variable Mission Mfg. (VMM)
3. Computerized Mfg. System (CMS)

## Four-Plan Concept of Manufacturing



CIM System discussed:

- Computer Numerical Control (CNC)
- Direct Numerical Control (DNC)
- Computer Process Control
- Computer Integrated Production Management
- Automated Inspection Methods
- Industrial Robots etc.

A CIM System consists of the following basic components:
I. Machine tools and related equipment
II. Material Handling System (MHS)
III. Computer Control System
IV. Human factor/labor

## CIMS Benefits:

1. Increased machine utilization
2. Reduced direct and indirect labor
3. Reduce mfg. lead time
4. Lower in process inventory
5. Scheduling flexibility
6. etc.

CIM refers to a production system that consists of:

1. A group of NC machines connected together by
2. An automated materials handling system
3. And operating under computer control

Why CIMS?
In Production Systems


1. Transfer Lines: is very efficient when producing "identical" parts in large volumes at high product rates.
2. Stand Alone: NC machine: are ideally suited for variations in work part configuration.

In Manufacturing Systems:


1. Special Mfg. System: the least flexible CIM system. It is designed to produce a
very limited number of different parts (2-8).
2. Mfg. Cell: the most flexible but generally has the lowest number of different parts manufactured in the cell would be between 40-80. Annual production rates rough from 200-500.
3. Flexible Mfg. System: A typical FMS will be used to process several part families with 4 to 100 different part numbers being the usual case.

## General FMS

## Conventional Approaches to Manufacturing

Conventional approaches to manufacturing have generally centered around machines laid out in logical arrangements in a manufacturing facility. These machine layouts are classified by:

1. Function - Machines organized by function will typically perform the same function, and the location of these departments relative to each other is normally


Machine layout by function.
arranged so as to minimize interdepartmental material handling. Workpiece produced in functional layout departments and factories are generally manufactured in small batches up to fifty pieces (a great variety of parts).
2. Line or flow layout - the arrangement of machines in the part processing order or sequence required. A transfer line is an example of a line layout. Parts progressively move from one machine to another in a line or flow layout by means of a roller conveyor or through manual material handling. Typically, one or very few different parts are produced on a line or flow type of layout, as all parts processed require the same processing sequence of operations. All machining is performed in one department, thereby minimizing interdepartmental material handling.


Line or flow departments

3. Cell - It combines the efficiencies of both layouts into a single multi-functional unit. It referred to as a group technology cell, each individual cell or department is comprised of different machines that may not be identical or even similar. Each cell is essentially a factory within a factory, and parts are grouped or arranged into families requiring the same type of processes, regardless of processing order. Cellular layouts are highly advantageous over both function and line machine layouts because they can eliminate complex material flow patterns and consolidate material movement from machine to machine within the cell.


Machine layout by cell based on part families to be processed

## Manufacturing Cell

## Four general categories:

1. Traditional stand-alone NC machine tool - is characterized as a limited-storage, automatic tool changer and is traditionally operated on a one-to-one machine to operator ratio. In many cased, stand-alone NC machine tools have been grouped together in a conventional part family manufacturing cell arrangement and operating on a one-to-one or two-to-one or three-to-one machine to operator ratio.
2. Single NC machine cell or mini-cell - is characterized by an automatic work changer with permanently assigned work pallets or a conveyor-robot arm system mounted to the front of the machine, plus the availability of bulk tool storage. There are many machines with a variety of options, such as automatic probing, broken tool detection, and high-pressure coolant control. The single NC machine cell is rapidly gaining in popularity, functionality, and affordability.
3. Integrated multi-machine cell - is made up of a multiplicity of metal-cutting machine tools, typically all of the same type, which have a queue of parts, either at the entry of the cell or in front of each machine. Multi-machine cells are either serviced by a material-handling robot or parts are palletized in a two- or three-machine, in-line system for progressive movement from one machining
station to another.
FMS - sometimes referred to as a flexible manufacturing cell (FMC), is characterized by multiple machines, automated random movement of palletize parts to and from processing stations, and central computer control with sophisticated command-driven software. The distinguishing characteristics of this cell are the automated flow of raw material to the cell, complete machining of the part, part washing, drying, and inspection with the cell, and removal of the finished part.

## I. Machine Tools \& Related Equipment

- Standard CNC machine tools
- Special purpose machine tools
- Tooling for these machines
- Inspection stations or special inspection probes used with the machine tool


## The Selection of Machine Tools

1. Part size
2. Part shape
3. Part variety
4. Product life cycle
5. Definition of function parts
6. Operations other than machining - assembly, inspection etc.

## II. Material Handling System

A. The primary work handling system - used to move parts between machine tools in the CIMS. It should meet the following requirements.
i). Compatibility with computer control
ii). Provide random, independent movement of palletized work parts between machine tools.
iii). Permit temporary storage or banking of work parts.
iv). Allow access to the machine tools for maintenance tool changing \& so on.
v). Interface with the secondary work handling system
vi). etc.
B. The secondary work handling system - used to present parts to the individual machine tools in the CIMS.
i). Same as A (i).
ii). Same as A (iii)
iii). Interface with the primary work handling system
iv). Provide for parts orientation \& location at each workstation for processing.
III. Computer Control System - Control functions of a firm and the supporting computing equipment
Control Function
Corporate Control

| Plant Floor |
| :--- |
| Control |
| Machine -Control |

## Control Loop of a Manufacturing System


IV. Functions of the computer in a manufacturing organization


## V. Functions of Computer in CIMS

## 1. Machine Control - CNC


2. Direct Numerical Control (DNC) - A manufacturing system in which a number of $\mathrm{m} / \mathrm{c}$ are controlled by a computer through direct connection $\&$ in real time.

## Consists of 4 basic elements:

- Central computer
- Bulk memory (NC program storage)
- Telecommunication line
- Machine tools (up to 100)


3. Production Control - This function includes decision on various parts onto the system.

## Decision are based on:

- red production rate/day for the various parts
- Number of raw work parts available
- Number of available pallets

4. Traffic \& Shuttle Control - Refers to the regulations of the primary \& secondary transportation systems which moves parts between workstation.
5. Work Handling System Monitoring - The computer must monitor the status of each cart \& /or pallet in the primary \& secondary handling system.
6. Tool Control

- Keeping track of the tool at each station
- Monitoring of tool life

7. System Performance Monitoring \& Reporting - The system computer can be programmed to generate various reports by the management on system performance.

- Utilization reports - summarize the utilization of individual workstation as well as overall average utilization of the system.
- Production reports - summarize weekly/daily quantities of parts produced from a CIMS (comparing scheduled production vs. actual production)
- Status reports - instantaneous report "snapshot" of the present conditions of the CIMS.
- Tool reports - may include a listing of missing tool, tool-life status etc.


## 8. Manufacturing data base

- Collection of independent data bases
- Centralized data base
- Interfaced data base
- Distributed data base





## Production Strategy

The production strategy used by manufacturers is based on several factors; the two most critical are customer lead time and manufacturing lead time.
Customer lead time identifies the maximum length of time that a typical customer is willing to wait for the delivery of a product after an order is placed.
Manufacturing lead time identifies the maximum length of time between the receipt of an order and the delivery of a finished product.
Manufacturing lead time and customer lead time must be matched. For example, when a new car with specific options is ordered from a dealer, the customer is willing to wait only a few weeks for delivery of the vehicle. As a result, automotive manufacturers must adopt a production strategy that permits the manufacturing lead-time to match the customer's needs.
The production strategies used to match the customer and manufacturer lead times are grouped into four categories:

1. Engineer to order (ETO)
2. Make to order (MTO)
3. Assemble to order (ATO)
4. Make to stock (MTS)

Engineer to Order
A manufacturer producing in this category has a product that is either in the first stage of the life-cycle curve or a complex product with a unique design produced in single-digit quantities. Examples of ETO include construction industry products (bridges, chemical plants, automotive production lines) and large products with special options that are stationary during production (commercial passenger aircraft, ships, high-voltage switchgear, steam turbines). Due to the nature of the product, the customer is willing to accept a long manufacturing lead time because the engineering design is part of the process.

Make to Order
The MTO technique assumes that all the engineering and design are complete and the production process is proven. Manufacturers use this strategy when the demand is
unpredictable and when the customer lead-time permits the production process to start on receipt of an order. New residential homes are examples of this production strategy. Some outline computer companies make personal computer to customer specifications, so they followed MTO specifications.

## Assemble to Order

The primary reason that manufacturers adopt the ATO strategy is that customer lead time is less than manufacturing lead time. An example from the automotive industry was used in the preceding section to describe this situation for line manufacturing systems. This strategy is used when the option mix for the products can be forecast statistically: for example, the percentage of four-door versus two-door automobiles assembled per week. In addition, the subassemblies and parts for the final product are carried in a finished components inventory, so the final assembly schedule is determined by the customer order. John Deere and General Motors are examples of companies using this production strategy.

## Make to Stock

MTS, is used for two reasons: (1) the customer lead time is less than the manufacturing lead time, (2) the product has a set configuration and few options so that the demand can be forecast accurately. If positive inventory levels (the store shelf is never empty) for a product is an order-winning criterion, this strategy is used. When this order-winning criterion is severe, the products are often stocked in distribution warehouses located in major population centers. This option is often the last phase of a product's life cycle and usually occurs at maximum production volume.

Manufacturing Enterprise (Organization)

- In most manufacturing organizations the functional blocks can be found as:
- A CIM implementation affects every part of an enterprise; as a result, every block in the organizational model is affected.



## Sales and Promotion

- The fundamental mission of sales and promotion (SP) is to create customers. To achieve this goal, nine internal functions are found in many companies: sales, customer service, advertising, product research and development, pricing, packaging, public relations, product distribution, and forecasting.
sales and promotion interfaces with several other areas in the business:
- The customer services interface supports three major customer functions: order entry, order changes, and order shipping and billing. The order change interface usually involves changes in product specifications, change in product quantity (ordered or available for shipment), and shipment dates and requirements.
- Sales and marketing provide strategic and production planning information to the finance and management group, product specification and customer feedback information to product design, and information for master production scheduling to the manufacturing planning and control group.
Product/Process Definition Engineering
- The unit includes product design, production engineering, and engineering release.
- The product design provides three primary functions: (1) product design and conceptualization, (2) material selection, and (3) design documentation.
- The production engineering area establishes three sets of standards: work, process, and quality.
- The engineering release area manages engineering change on every production part in the enterprise. Engineering release has the responsibility of securing approvals from departments across the enterprise for changes made in the product or production process.
Manufacturing Planning and Control (MPC)
- The manufacturing planning and control unit has a formal data and information interface with several other units and departments in the enterprise.
- The MPC unit has responsibility for:

1. Setting the direction for the enterprise by translating the management plan into manufacturing terms. The translation is smooth if order-winning criteria were used to develop the management plan.
2. Providing detailed planning for material flow and capacity to support the overall plan.
3. Executing these plans through detailed shop scheduling and purchasing action.
MPC Model for Information Flow


## Shop Floor

- Shop floor activity often includes job planning and reporting, material movement, manufacturing process, plant floor control, and quality control.
- Interfaces with the shop floor unit are illustrated.



## Support Organization

- The support organizations, indicated vary significantly from firm to firm.
- The functions most often included are security, personnel, maintenance, human resource development, and computer services.
- Basically, the support organization is responsible for all of the functions not provided by the other model elements.
Production Sequence :one possibility for the flow required to bring a product to a customer



# OPTIMIZATION <br> An introduction 

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First draft - September 2002
Last revision - September 2006

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## Chapter 1

## Introduction

### 1.1 Introduction

Optimization is the act of achieving the best possible result under given circumstances. In design, construction, maintenance, ..., engineers have to take decisions. The goal of all such decisions is either to minimize effort or to maximize benefit.
The effort or the benefit can be usually expressed as a function of certain design variables. Hence, optimization is the process of finding the conditions that give the maximum or the minimum value of a function.
It is obvious that if a point $x^{\star}$ corresponds to the minimum value of a function $f(x)$, the same point corresponds to the maximum value of the function $-f(x)$. Thus, optimization can be taken to be minimization.
There is no single method available for solving all optimization problems efficiently. Hence, a number of methods have been developed for solving different types of problems.
Optimum seeking methods are also known as mathematical programming techniques, which are a branch of operations research. Operations research is coarsely composed of the following areas.

- Mathematical programming methods. These are useful in finding the minimum of a function of several variables under a prescribed set of constraints.
- Stochastic process techniques. These are used to analyze problems which are described by a set of random variables of known distribution.
- Statistical methods. These are used in the analysis of experimental data and in the construction of empirical models.

These lecture notes deal mainly with the theory and applications of mathematical programming methods. Mathematical programming is a vast area of mathematics and engineering. It includes

- calculus of variations and optimal control;
- linear, quadratic and non-linear programming;
- geometric programming;
- integer programming;
- network methods (PERT);
- game theory.

The existence of optimization can be traced back to Newton, Lagrange and Cauchy. The development of differential methods for optimization was possible because of the contribution of Newton and Leibnitz. The foundations of the calculus of variations were laid by Bernoulli, Euler, Lagrange and Weierstrasse. Constrained optimization was first studied by Lagrange and the notion of descent was introduced by Cauchy.

Despite these early contributions, very little progress was made till the 20th century, when computer power made the implementation of optimization procedures possible and this in turn stimulated further research methods.
The major developments in the area of numerical methods for unconstrained optimization have been made in the UK. These include the development of the simplex method (Dantzig, 1947), the principle of optimality (Bellman, 1957), necessary and sufficient conditions of optimality (Kuhn and Tucker, 1951).
Optimization in its broadest sense can be applied to solve any engineering problem, e.g.

- design of aircraft for minimum weight;
- optimal (minimum time) trajectories for space missions;
- minimum weight design of structures for earthquake;
- optimal design of electric networks;
- optimal production planning, resources allocation, scheduling;
- shortest route;
- design of optimum pipeline networks;
- minimum processing time in production systems;
- optimal control.


### 1.2 Statement of an optimization problem

An optimization, or a mathematical programming problem can be stated as follows. Find

$$
x=\left(x^{1}, x^{2}, \ldots ., x^{n}\right)
$$

which minimizes

$$
f(x)
$$

subject to the constraints

$$
\begin{equation*}
g_{j}(x) \leq 0 \tag{1.1}
\end{equation*}
$$

for $j=1, \ldots, m$, and

$$
\begin{equation*}
l_{j}(x)=0 \tag{1.2}
\end{equation*}
$$

for $j=1, \ldots, p$.
The variable $x$ is called the design vector, $f(x)$ is the objective function, $g_{j}(x)$ are the inequality constraints and $l_{j}(x)$ are the equality constraints. The number of variables $n$ and the number of constraints $p+m$ need not be related. If $p+m=0$ the problem is called an unconstrained optimization problem.


Figure 1.1: Feasible region in a two-dimensional design space. Only inequality constraints are present.

### 1.2.1 Design vector

Any system is described by a set of quantities, some of which are viewed as variables during the design process, and some of which are preassigned parameters or are imposed by the environment. All the quantities that can be treated as variables are called design or decision variables, and are collected in the design vector $x$.

### 1.2.2 Design constraints

In practice, the design variables cannot be selected arbitrarily, but have to satisfy certain requirements. These restrictions are called design constraints. Design constraints may represent limitation on the performance or behaviour of the system or physical limitations. Consider, for example, an optimization problem with only inequality constraints, i.e. $g_{j}(x) \leq 0$. The set of values of $x$ that satisfy the equations $g_{j}(x)=0$ forms a hypersurface in the design space, which is called constraint surface. In general, if $n$ is the number of design variables, the constraint surface is an $n-1$ dimensional surface. The constraint surface divides the design space into two regions: one in which $g_{j}(x)<0$ and one in which $g_{j}(x)>0$. The points $x$ on the constraint surface satisfy the constraint critically, whereas the points $x$ such that $g_{j}(x)>0$, for some $j$, are infeasible, i.e. are unacceptable, see Figure 1.1.

### 1.2.3 Objective function

The classical design procedure aims at finding an acceptable design, i.e. a design which satisfies the constraints. In general there are several acceptable designs, and the purpose


Figure 1.2: Design space, objective functions surfaces, and optimum point.
of the optimization is to single out the best possible design. Thus, a criterion has to be selected for comparing different designs. This criterion, when expressed as a function of the design variables, is known as objective function. The objective function is in general specified by physical or economical considerations. However, the selection of an objective function is not trivial, because what is the optimal design with respect to a certain criterion may be unacceptable with respect to another criterion. Typically there is a trade off performance-cost, or performance-reliability, hence the selection of the objective function is one of the most important decisions in the whole design process. If more than one criterion has to be satisfied we have a multiobjective optimization problem, that may be approximately solved considering a cost function which is a weighted sum of several objective functions.

Given an objective function $f(x)$, the locus of all points $x$ such that $f(x)=c$ forms a hypersurface. For each value of $c$ there is a different hypersurface. The set of all these surfaces are called objective function surfaces.

Once the objective function surfaces are drawn, together with the constraint surfaces, the optimization problem can be easily solved, at least in the case of a two dimensional decision space, as shown in Figure 1.2. If the number of decision variables exceeds two or three, this graphical approach is not viable and the problem has to be solved as a mathematical problem. Note however that more general problems have similar geometrical properties of two or three dimensional problems.


Figure 1.3: Electrical bridge network.

### 1.3 Classification of optimization problems

Optimization problem can be classified in several ways.

- Existence of constraints. An optimization problem can be classified as a constrained or an unconstrained one, depending upon the presence or not of constraints.
- Nature of the equations. Optimization problems can be classified as linear, quadratic, polynomial, non-linear depending upon the nature of the objective functions and the constraints. This classification is important, because computational methods are usually selected on the basis of such a classification, i.e. the nature of the involved functions dictates the type of solution procedure.
- Admissible values of the design variables. Depending upon the values permitted for the design variables, optimization problems can be classified as integer or real valued, and deterministic or stochastic.


### 1.4 Examples

Example 1 A travelling salesman has to cover $n$ towns. He plans to start from a particular town numbered 1 , visit each one of the other $n-1$ towns, and return to the town 1 . The distance between town $i$ and $j$ is given by $d_{i j}$. How should he select the sequence in which the towns are visited to minimize the total distance travelled?

Example 2 The bridge network in Figure 1.3 consists of five resistors $R_{i}, i=1, \ldots, 5$. Let $I_{i}$ be the current through the resistance $R_{i}$, find the values of $R_{i}$ so that the total dissipated power is minimum. The current $I_{i}$ can vary between the lower limit $\underline{\mathrm{I}}_{i}$ and the upper limit $\bar{I}_{i}$ and the voltage drop $V_{i}=R_{i} I_{i}$ must be equal to a constant $c_{i}$.

Example 3 A manufacturing firm produces two products, A and B, using two limited resources, 1 and 2. The maximum amount of resource 1 available per week is 1000 and the

| Article type | $w_{i}$ | $v_{i}$ | $c_{i}$ |
| :---: | :---: | :---: | :---: |
| 1 | 4 | 9 | 5 |
| 2 | 8 | 7 | 6 |
| 3 | 2 | 4 | 3 |

Table 1.1: Properties of the articles to load.
maximum amount of resource 2 is 250 . The production of one unit of A requires 1 unit of resource 1 and $1 / 5$ unit of resource 2 . The production of one unit of $B$ requires $1 / 2$ unit of resource 1 and $1 / 2$ unit of resource 2 . The unit cost of resource 1 is $1-0.0005 u_{1}$, where $u_{1}$ is the number of units of resource 1 used. The unit cost of resource 2 is $3 / 4-0.0001 u_{2}$, where $u_{2}$ is the number of units of resource 2 used. The selling price of one unit of A is

$$
2-0.005 x_{A}-0.0001 x_{B}
$$

and the selling price of one unit of $B$ is

$$
4-0.002 x_{A}-0.01 x_{B},
$$

where $x_{A}$ and $x_{B}$ are the number of units of A and B sold. Assuming that the firm is able to sell all manufactured units, maximize the weekly profit.

Example 4 A cargo load is to be prepared for three types of articles. The weight, $w_{i}$, volume, $v_{i}$, and value, $c_{i}$, of each article is given in Table 1.1.
Find the number of articles $x_{i}$ selected from type $i$ so that the total value of the cargo is maximized. The total weight and volume of the cargo cannot exceed 2000 and 2500 units respectively.

Example 5 There are two types of gas molecules in a gaseous mixture at equilibrium. It is known that the Gibbs free energy

$$
G(x)=c_{1} x^{1}+c_{2} x^{2}+x^{1} \log \left(x^{1} / x_{T}\right)+x^{2} \log \left(x^{2} / x_{T}\right),
$$

with $x_{T}=x^{1}+x^{2}$ and $c_{1}, c_{2}$ known parameters depending upon the temperature and pressure of the mixture, has to be minimum in these conditions. The minimization of $G(x)$ is also subject to the mass balance equations:

$$
x^{1} a_{i 1}+x^{2} a_{i 2}=b_{i},
$$

for $i=1, \ldots, m$, where $m$ is the number of atomic species in the mixture, $b_{i}$ is the total weight of atoms of type $i$, and $a_{i j}$ is the number of atoms of type $i$ in the molecule of type $j$. Show that the problem of determining the equilibrium of the mixture can be posed as an optimization problem.

## Chapter 2

## Unconstrained optimization

### 2.1 Introduction

Several engineering, economic and planning problems can be posed as optimization problems, i.e. as the problem of determining the points of minimum of a function (possibly in the presence of conditions on the decision variables). Moreover, also numerical problems, such as the problem of solving systems of equations or inequalities, can be posed as an optimization problem.
We start with the study of optimization problems in which the decision variables are defined in $\mathbb{R}^{n}$ : unconstrained optimization problems. More precisely we study the problem of determining local minima for differentiable functions. Although these methods are seldom used in applications, as in real problems the decision variables are subject to constraints, the techniques of unconstrained optimization are instrumental to solve more general problems: the knowledge of good methods for local unconstrained minimization is a necessary pre-requisite for the solution of constrained and global minimization problems. The methods that will be studied can be classified from various points of view. The most interesting classification is based on the information available on the function to be optimized, namely

- methods without derivatives (direct search, finite differences);
- methods based on the knowledge of the first derivatives (gradient, conjugate directions, quasi-Newton);
- methods based on the knowledge of the first and second derivatives (Newton).


### 2.2 Definitions and existence conditions

Consider the optimization problem:
Problem 1 Minimize

$$
f(x) \quad \text { subject to } x \in \mathcal{F}
$$

in which $f: \mathbb{R}^{n} \rightarrow \mathbb{R}$ and ${ }^{1} \mathcal{F} \subset \mathbb{R}^{n}$.
With respect to this problem we introduce the following definitions.
Definition $1 A$ point $x \in \mathcal{F}$ is a global minimum ${ }^{2}$ for the Problem 1 if

$$
f(x) \leq f(y)
$$

for all $y \in \mathcal{F}$.
A point $x \in \mathcal{F}$ is a strict (or isolated) global minimum (or minimiser) for the Problem 1 if

$$
f(x)<f(y)
$$

[^2]for all $y \in \mathcal{F}$ and $y \neq x$.
A point $x \in \mathcal{F}$ is a local minimum (or minimiser) for the Problem 1 if there exists $\rho>0$ such that
$$
f(x) \leq f(y)
$$
for all $y \in \mathcal{F}$ such that $\|y-x\|<\rho$.
A point $x \in \mathcal{F}$ is a strict (or isolated) local minimum (or minimiser) for the Problem 1 if there exists $\rho>0$ such that
$$
f(x)<f(y)
$$
for all $y \in \mathcal{F}$ such that $\|y-x\|<\rho$ and $y \neq x$.
Definition 2 If $x \in \mathcal{F}$ is a local minimum for the Problem 1 and if $x$ is in the interior of $\mathcal{F}$ then $x$ is an unconstrained local minimum of $f$ in $\mathcal{F}$.

The following result provides a sufficient, but not necessary, condition for the existence of a global minimum for Problem 1.

Proposition 1 Let $f: \mathbb{R}^{n} \rightarrow \mathbb{R}$ be a continuous function and let $\mathcal{F} \subset \mathbb{R}^{n}$ be a compact set ${ }^{3}$. Then there exists a global minimum of $f$ in $\mathcal{F}$.

In unconstrained optimization problems the set $\mathcal{F}$ coincides with $\mathbb{R}^{n}$, hence the above statement cannot be used to establish the existence of global minima. To address the existence problem it is necessary to consider the structure of the level sets of the function $f$. See also Section 1.2.3.

Definition 3 Let $f: \mathbb{R}^{n} \rightarrow \mathbb{R}$. A level set of $f$ is any non-empty set described by

$$
\mathcal{L}(\alpha)=\left\{x \in \mathbb{R}^{n}: f(x) \leq \alpha\right\},
$$

with $\alpha \in \mathbb{R}$.
For convenience, if $x_{0} \in \mathbb{R}^{n}$ we denote with $\mathcal{L}_{0}$ the level set $\mathcal{L}\left(f\left(x_{0}\right)\right)$. Using the concept of level sets it is possible to establish a simple sufficient condition for the existence of global solutions for an unconstrained optimization problem.

Proposition 2 Let $f: \mathbb{R}^{n} \rightarrow \mathbb{R}$ be a continuous function. Assume there exists $x_{0} \in \mathbb{R}^{n}$ such that the level set $\mathcal{L}_{0}$ is compact. Then there exists a point of global minimum of $f$ in $\mathbb{R}^{n}$.

Proof. By Proposition 1 there exists a global minimum $x_{\star}$ of $f$ in $\mathcal{L}_{0}$, i.e. $f\left(x_{\star}\right) \leq f(x)$ for all $x \in \mathcal{L}_{0}$. However, if $x \notin \mathcal{L}_{0}$ then $f(x)>f\left(x_{0}\right) \geq f\left(x_{\star}\right)$, hence $x_{\star}$ is a global minimum of $f$ in $\mathbb{R}^{n}$.

It is obvious that the structure of the level sets of the function $f$ plays a fundamental role in the solution of Problem 1. The following result provides a necessary and sufficient condition for the compactness of all level sets of $f$.

[^3]Proposition 3 Let $f: \mathbb{R}^{n} \rightarrow \mathbb{R}$ be a continuous function. All level sets of $f$ are compact if and only if for any sequence $\left\{x_{k}\right\}$ one has

$$
\lim _{k \rightarrow \infty}\left\|x_{k}\right\|=\infty \quad \Rightarrow \quad \lim _{k \rightarrow \infty} f\left(x_{k}\right)=\infty .
$$

Remark. In general $x_{k} \in \mathbb{R}^{n}$, namely

$$
x_{k}=\left[\begin{array}{c}
x_{k}^{1} \\
x_{k}^{2} \\
\vdots \\
x_{k}^{n}
\end{array}\right]
$$

i.e. we use superscripts to denote components of a vector.

A function that satisfies the condition of the above proposition is said to be radially unbounded.

Proof. We only prove the necessity. Suppose all level sets of $f$ are compact. Then, proceeding by contradiction, suppose there exist a sequence $\left\{x_{k}\right\}$ such that $\lim _{k \rightarrow \infty}\left\|x_{k}\right\|=$ $\infty$ and a number $\gamma>0$ such that $f\left(x_{k}\right) \leq \gamma<\infty$ for all $k$. As a result

$$
\left\{x_{k}\right\} \subset \mathcal{L}(\gamma)
$$

However, by compactness of $\mathcal{L}(\gamma)$ it is not possible that $\lim _{k \rightarrow \infty}\left\|x_{k}\right\|=\infty$.

Definition 4 Let $f: \mathbb{R}^{n} \rightarrow \mathbb{R}$. A vector $d \in \mathbb{R}^{n}$ is said to be a descent direction for $f$ in $x_{\star}$ if there exists $\delta>0$ such that

$$
f\left(x_{\star}+\lambda d\right)<f\left(x_{\star}\right),
$$

for all $\lambda \in(0, \delta)$.
If the function $f$ is differentiable it is possible to give a simple condition guaranteeing that a certain direction is a descent direction.

Proposition 4 Let $f: \mathbb{R}^{n} \rightarrow \mathbb{R}$ and assume ${ }^{4} \nabla f$ exists and is continuous. Let $x_{\star}$ and $d$ be given. Then, if $\nabla f\left(x_{\star}\right)^{\prime} d<0$ the direction $d$ is a descent direction for $f$ at $x_{\star}$.

Proof. Note that $\nabla f\left(x_{\star}\right)^{\prime} d$ is the directional derivative of $f$ (which is differentiable by hypothesis) at $x_{\star}$ along $d$, i.e.

$$
\nabla f\left(x_{\star}\right)^{\prime} d=\lim _{\lambda \rightarrow 0^{+}} \frac{f\left(x_{\star}+\lambda d\right)-f\left(x_{\star}\right)}{\lambda},
$$

[^4]

Figure 2.1: Geometrical interpretation of the anti-gradient.
and this is negative by hypothesis. As a result, for $\lambda>0$ and sufficiently small

$$
f\left(x_{\star}+\lambda d\right)-f\left(x_{\star}\right)<0,
$$

hence the claim.
The proposition establishes that if $\nabla f\left(x_{\star}\right)^{\prime} d<0$ then for sufficiently small positive displacements along $d$ and starting at $x_{\star}$ the function $f$ is decreasing. It is also obvious that if $\nabla f\left(x_{\star}\right)^{\prime} d>0, d$ is a direction of ascent, i.e. the function $f$ is increasing for sufficiently small positive displacements from $x_{\star}$ along $d$. If $\nabla f\left(x_{\star}\right)^{\prime} d=0, d$ is orthogonal to $\nabla f\left(x_{\star}\right)$ and it is not possible to establish, without further knowledge on the function $f$, what is the nature of the direction $d$.
From a geometrical point of view (see also Figure 2.1), the sign of the directional derivative $\nabla f\left(x_{\star}\right)^{\prime} d$ gives information on the angle between $d$ and the direction of the gradient at $x_{\star}$, provided $\nabla f\left(x_{\star}\right) \neq 0$. If $\nabla f\left(x_{\star}\right)^{\prime} d>0$ the angle between $\nabla f\left(x_{\star}\right)$ and $d$ is acute. If $\nabla f\left(x_{\star}\right)^{\prime} d<0$ the angle between $\nabla f\left(x_{\star}\right)$ and $d$ is obtuse. Finally, if $\nabla f\left(x_{\star}\right)^{\prime} d=0$, and $\nabla f\left(x_{\star}\right) \neq 0, \nabla f\left(x_{\star}\right)$ and $d$ are orthogonal. Note that the gradient $\nabla f\left(x_{\star}\right)$, if it is not identically zero, is a direction orthogonal to the level surface $\left\{x: f(x)=f\left(x_{\star}\right)\right\}$ and it is a direction of ascent, hence the anti-gradient $-\nabla f\left(x_{\star}\right)$ is a descent direction.

Remark. The scalar product $x^{\prime} y$ between the two vectors $x$ and $y$ can be used to define the angle between $x$ and $y$. For, define the angle between $x$ and $y$ as the number $\theta \in[0, \pi]$ such that ${ }^{5}$

$$
\cos \theta=\frac{x^{\prime} y}{\|x\|_{E}\|y\|_{E}}
$$

If $x^{\prime} y=0$ one has $\cos \theta=0$ and the vectors are orthogonal, whereas if $x$ and $y$ have the same direction, i.e. $x=\lambda y$ with $\lambda>0, \cos \theta=1$.

[^5]We are now ready to state and prove some necessary conditions and some sufficient conditions for a local minimum.

Theorem 1 [First order necessary condition] Let $f: \mathbb{R}^{n} \rightarrow \mathbb{R}$ and assume $\nabla f$ exists and is continuous. The point $x_{\star}$ is a local minimum of $f$ only if

$$
\nabla f\left(x_{\star}\right)=0 .
$$

Remark. A point $x_{\star}$ such that $\nabla f\left(x_{\star}\right)=0$ is called a stationary point of $f$.
Proof. If $\nabla f\left(x_{\star}\right) \neq 0$ the direction $d=-\nabla f\left(x_{\star}\right)$ is a descent direction. Therefore, in a neighborhood of $x_{\star}$ there is a point $x_{\star}+\lambda d=x_{\star}-\lambda \nabla f\left(x_{\star}\right)$ such that

$$
f\left(x_{\star}-\lambda \nabla f\left(x_{\star}\right)\right)<f\left(x_{\star}\right),
$$

and this contradicts the hypothesis that $x_{\star}$ is a local minimum.

Theorem 2 [Second order necessary condition] Let $f: \mathbb{R}^{n} \rightarrow \mathbb{R}$ and assume ${ }^{6} \nabla^{2} f$ exists and is continuous. The point $x_{\star}$ is a local minimum of $f$ only if

$$
\nabla f\left(x_{\star}\right)=0
$$

and

$$
x^{\prime} \nabla^{2} f\left(x_{\star}\right) x \geq 0
$$

for all $x \in \mathbb{R}^{n}$.
Proof. The first condition is a consequence of Theorem 1. Note now that, as $f$ is two times differentiable, for any $x \neq x_{\star}$ one has

$$
f\left(x_{\star}+\lambda x\right)=f\left(x_{\star}\right)+\lambda \nabla f\left(x_{\star}\right)^{\prime} x+\frac{1}{2} \lambda^{2} x^{\prime} \nabla^{2} f\left(x_{\star}\right) x+\beta\left(x_{\star}, \lambda x\right),
$$

where

$$
\lim _{\lambda \rightarrow 0} \frac{\beta\left(x_{\star}, \lambda x\right)}{\lambda^{2}\|x\|^{2}}=0
$$

or what is the same (note that $x$ is fixed)

$$
\lim _{\lambda \rightarrow 0} \frac{\beta\left(x_{\star}, \lambda x\right)}{\lambda^{2}}=0
$$

${ }^{6}$ We denote with $\nabla^{2} f$ the Hessian matrix of the function $f$, i.e.

$$
\left[\begin{array}{ccc}
\frac{\partial^{2} f}{\partial x^{1} \partial x^{1}} & \cdots & \frac{\partial^{2} f}{\partial x^{1} \partial x^{n}} \\
\vdots & \ddots & \vdots \\
\frac{\partial^{n} f}{\partial x^{n} \partial x^{1}} & \cdots & \frac{\partial^{2} f}{\partial x^{n} \partial x^{n}}
\end{array}\right]
$$

Note that $\nabla^{2} f$ is a square matrix and that, under suitable regularity conditions, the Hessian matrix is symmetric.

Moreover, the condition $\nabla f\left(x_{\star}\right)=0$ yields

$$
\begin{equation*}
\frac{f\left(x_{\star}+\lambda x\right)-f\left(x_{\star}\right)}{\lambda^{2}}=\frac{1}{2} x^{\prime} \nabla^{2} f\left(x_{\star}\right) x+\frac{\beta\left(x_{\star}, \lambda x\right)}{\lambda^{2}} . \tag{2.1}
\end{equation*}
$$

However, as $x_{\star}$ is a local minimum, the left hand side of equation (2.1) must be nonnegative for all $\lambda$ sufficiently small, hence

$$
\frac{1}{2} x^{\prime} \nabla^{2} f\left(x_{\star}\right) x+\frac{\beta\left(x_{\star}, \lambda x\right)}{\lambda^{2}} \geq 0
$$

and

$$
\lim _{\lambda \rightarrow 0}\left(\frac{1}{2} x^{\prime} \nabla^{2} f\left(x_{\star}\right) x+\frac{\beta\left(x_{\star}, \lambda x\right)}{\lambda^{2}}\right)=\frac{1}{2} x^{\prime} \nabla^{2} f\left(x_{\star}\right) x \geq 0
$$

which proves the second condition.

Theorem 3 (Second order sufficient condition) Let $f: \mathbb{R}^{n} \rightarrow \mathbb{R}$ and assume $\nabla^{2} f$ exists and is continuous. The point $x_{\star}$ is a strict local minimum of $f$ if

$$
\nabla f\left(x_{\star}\right)=0
$$

and

$$
x^{\prime} \nabla^{2} f\left(x_{\star}\right) x>0
$$

for all non-zero $x \in \mathbb{R}^{n}$.
Proof. To begin with, note that as $\nabla^{2} f\left(x_{\star}\right)>0$ and $\nabla^{2} f$ is continuous, then there is a neighborhood $\Omega$ of $x_{\star}$ such that for all $y \in \Omega$

$$
\nabla^{2} f(y)>0
$$

Consider now the Taylor series expansion of $f$ around the point $x_{\star}$, i.e.

$$
f(y)=f\left(x_{\star}\right)+\nabla f\left(x_{\star}\right)^{\prime}\left(y-x_{\star}\right)+\frac{1}{2}\left(y-x_{\star}\right)^{\prime} \nabla^{2} f(\xi)\left(y-x_{\star}\right),
$$

where $\xi=x_{\star}+\theta\left(y-x_{\star}\right)$, for some $\theta \in[0,1]$. By the first condition one has

$$
f(y)=f\left(x_{\star}\right)+\frac{1}{2}\left(y-x_{\star}\right)^{\prime} \nabla^{2} f(\xi)\left(y-x_{\star}\right),
$$

and, for any $y \in \Omega$ such that $y \neq x_{\star}$,

$$
f(y)>f\left(x_{\star}\right),
$$

which proves the claim.
The above results can be easily modified to derive necessary conditions and sufficient conditions for a local maximum. Moreover, if $x_{\star}$ is a stationary point and the Hessian matrix


Figure 2.2: A saddle point in $\mathbb{R}^{2}$.
$\nabla^{2} f\left(x_{\star}\right)$ is indefinite, the point $x_{\star}$ is neither a local minimum neither a local maximum. Such a point is called a saddle point (see Figure 2.2 for a geometrical illustration).
If $x_{\star}$ is a stationary point and $\nabla^{2} f\left(x_{\star}\right)$ is semi-definite it is not possible to draw any conclusion on the point $x_{\star}$ without further knowledge on the function $f$. Nevertheless, if $n=1$ and the function $f$ is infinitely times differentiable it is possible to establish the following necessary and sufficient condition.

Proposition 5 Let $f: \mathbb{R} \rightarrow \mathbb{R}$ and assume $f$ is infinitely times differentiable. The point $x_{\star}$ is a local minimum if and only if there exists an even integer $r>1$ such that

$$
\frac{d^{k} f\left(x_{\star}\right)}{d x^{k}}=0
$$

for $k=1,2, \ldots, r-1$ and

$$
\frac{d^{r} f\left(x_{\star}\right)}{d x^{r}}>0 .
$$

Necessary and sufficient conditions for $n>1$ can be only derived if further hypotheses on the function $f$ are added, as shown for example in the following fact.

Proposition 6 (Necessary and sufficient condition for convex functions) Let $f$ : $\mathbb{R}^{n} \rightarrow \mathbb{R}$ and assume $\nabla f$ exists and it is continuous. Suppose $f$ is convex, i.e.

$$
\begin{equation*}
f(y)-f(x) \geq \nabla f(x)^{\prime}(y-x) \tag{2.2}
\end{equation*}
$$

for all $x \in \mathbb{R}^{n}$ and $y \in \mathbb{R}^{n}$. The point $x_{\star}$ is a global minimum if and only if $\nabla f\left(x_{\star}\right)=0$.

Proof. The necessity is a consequence of Theorem 1. For the sufficiency note that, by equation (2.2), if $\nabla f\left(x_{\star}\right)=0$ then

$$
f(y) \geq f\left(x_{\star}\right)
$$

for all $y \in \mathbb{R}^{n}$.
From the above discussion it is clear that to establish the property that $x_{\star}$, satisfying $\nabla f\left(x_{\star}\right)=0$, is a global minimum it is enough to assume that the function $f$ has the following property: for all $x$ and $y$ such that

$$
\nabla f(x)^{\prime}(y-x) \geq 0
$$

one has

$$
f(y) \geq f(x)
$$

A function $f$ satisfying the above property is said pseudo-convex. Note that a differentiable convex function is also pseudo-convex, but the opposite is not true. For example, the function $x+x^{3}$ is pseudo-convex but it is not convex. Finally, if $f$ is strictly convex or strictly pseudo-convex the global minimum (if it exists) is also unique.

### 2.3 General properties of minimization algorithms

Consider the problem of minimizing the function $f: \mathbb{R}^{n} \rightarrow \mathbb{R}$ and suppose that $\nabla f$ and $\nabla^{2} f$ exist and are continuous. Suppose that such a problem has a solution, and moreover that there exists $x_{0}$ such that the level set

$$
\mathcal{L}\left(f\left(x_{0}\right)\right)=\left\{x \in \mathbb{R}^{n}: f(x) \leq f\left(x_{0}\right)\right\}
$$

is compact.
General unconstrained minimization algorithms allow only to determine stationary points of $f$, i.e. to determine points in the set

$$
\Omega=\left\{x \in \mathbb{R}^{n}: \nabla f(x)=0\right\} .
$$

Moreover, for almost all algorithms, it is possible to exclude that the points of $\Omega$ yielded by the algorithm are local maxima. Finally, some algorithms yield points of $\Omega$ that satisfy also the second order necessary conditions.

### 2.3.1 General unconstrained minimization algorithm

An algorithm for the solution of the considered minimization problem is a sequence $\left\{x_{k}\right\}$, obtained starting from an initial point $x_{0}$, having some convergence properties in relation with the set $\Omega$. Most of the algorithms that will be studied in this notes can be described in the following general way.

1. Fix a point $x_{0} \in \mathbb{R}^{n}$ and set $k=0$.
2. If $x_{k} \in \Omega$ STOP.
3. Compute a direction of research $d_{k} \in \mathbb{R}^{n}$.
4. Compute a step $\alpha_{k} \in \mathbb{R}$ along $d_{k}$.
5. Let $x_{k+1}=x_{k}+\alpha_{k} d_{k}$. Set $k=k+1$ and go back to 2 .

The existing algorithms differ in the way the direction of research $d_{k}$ is computed and on the criteria used to compute the step $\alpha_{k}$. However, independently from the particular selection, it is important to study the following issues:

- the existence of accumulation points for the sequence $\left\{x_{k}\right\}$;
- the behavior of such accumulation points in relation with the set $\Omega$;
- the speed of convergence of the sequence $\left\{x_{k}\right\}$ to the points of $\Omega$.


### 2.3.2 Existence of accumulation points

To make sure that any subsequence of $\left\{x_{k}\right\}$ has an accumulation point it is necessary to assume that the sequence $\left\{x_{k}\right\}$ remains bounded, i.e. that there exists $M>0$ such that $\left\|x_{k}\right\|<M$ for any $k$. If the level set $\mathcal{L}\left(f\left(x_{0}\right)\right)$ is compact, the above condition holds if $\left\{x_{k}\right\} \in \mathcal{L}\left(f\left(x_{0}\right)\right)$. This property, in turn, is guaranteed if

$$
f\left(x_{k+1}\right)<f\left(x_{k}\right),
$$

for any $k$ such that $x_{k} \notin \Omega$. The algorithms that satisfy this property are denominated descent methods. For such methods, if $\mathcal{L}\left(f\left(x_{0}\right)\right)$ is compact and if $\nabla f$ is continuous one has

- $\left\{x_{k}\right\} \in \mathcal{L}\left(f\left(x_{0}\right)\right)$ and any subsequence of $\left\{x_{k}\right\}$ admits a subsequence converging to a point of $\mathcal{L}\left(f\left(x_{0}\right)\right)$;
- the sequence $\left\{f\left(x_{k}\right)\right\}$ has a limit, i.e. there exists $\bar{f} \in \mathbb{R}$ such that

$$
\lim _{k \rightarrow \infty} f\left(x_{k}\right)=\bar{f}
$$

- there always exists an element of $\Omega$ in $\mathcal{L}\left(f\left(x_{0}\right)\right)$. In fact, as $f$ has a minimum in $\mathcal{L}\left(f\left(x_{0}\right)\right)$, this minimum is also a minimum of $f$ in $\mathbb{R}^{n}$. Hence, by the assumptions of $\nabla f$, such a minimum must be a point of $\Omega$.

Remark. To guarantee the descent property it is necessary that the research directions $d_{k}$ be directions of descent. This is true if

$$
\nabla f\left(x_{k}\right)^{\prime} d_{k}<0
$$

for all $k$. Under this condition there exists an interval $\left(0, \alpha_{\star}\right]$ such that

$$
f\left(x_{k}+\alpha d_{k}\right)<f\left(x_{k}\right),
$$

for any $\alpha \in\left(0, \alpha_{\star}\right]$.
Remark. The existence of accumulation points for the sequence $\left\{x_{k}\right\}$ and the convergence of the sequence $\left\{f\left(x_{k}\right)\right\}$ do not guarantee that the accumulation points of $\left\{x_{k}\right\}$ are local minima of $f$ or stationary points. To obtain this property it is necessary to impose further restrictions on the research directions $d_{k}$ and on the steps $\alpha_{k}$.

### 2.3.3 Condition of angle

The condition which is in general imposed on the research directions $d_{k}$ is the so-called condition of angle, that can be stated as follows.

Condition 1 There exists $\epsilon>0$, independent from $k$, such that

$$
\nabla f\left(x_{k}\right)^{\prime} d_{k} \leq-\epsilon\left\|\nabla f\left(x_{k}\right)\right\|\left\|d_{k}\right\|,
$$

for any $k$.
From a geometric point of view the above condition implies that the cosine of the angle between $d_{k}$ and $-\nabla f\left(x_{k}\right)$ is larger than a certain quantity. This condition is imposed to avoid that, for some $k$, the research direction is orthogonal to the direction of the gradient. Note moreover that, if the angle condition holds, and if $\nabla f\left(x_{k}\right) \neq 0$ then $d_{k}$ is a descent direction. Finally, if $\nabla f\left(x_{k}\right) \neq 0$, it is always possible to find a direction $d_{k}$ such that the angle condition holds. For example, the direction $d_{k}=-\nabla f\left(x_{k}\right)$ is such that the angle condition is satisfied with $\epsilon=1$.

Remark. Let $\left\{B_{k}\right\}$ be a sequence of matrices such that

$$
m I \leq B_{k} \leq M I,
$$

for some $0<m<M$, and for any $k$, and consider the directions

$$
d_{k}=-B_{k} \nabla f\left(x_{k}\right) .
$$

Then a simple computation shows that the angle condition holds with $\epsilon=m / M$. $\diamond$
The angle condition imposes a constraint only on the research directions $d_{k}$. To make sure that the sequence $\left\{x_{k}\right\}$ converges to a point in $\Omega$ it is necessary to impose further conditions on the step $\alpha_{k}$, as expressed in the following statements.

Theorem 4 Let $\left\{x_{k}\right\}$ be the sequence obtained by the algorithm

$$
x_{k+1}=x_{k}+\alpha_{k} d_{k},
$$

for $k \geq 0$. Assume that
(H1) $\nabla f$ is continuous and the level set $\mathcal{L}\left(f\left(x_{0}\right)\right)$ is compact.
(H2) There exists $\epsilon>0$ such that

$$
\nabla f\left(x_{k}\right)^{\prime} d_{k} \leq-\epsilon\left\|\nabla f\left(x_{k}\right)\right\|\left\|d_{k}\right\|,
$$

for any $k \geq 0$.
(H3) $f\left(x_{k+1}\right)<f\left(x_{k}\right)$ for any $k \geq 0$.
(H4) The property

$$
\lim _{k \rightarrow \infty} \frac{\nabla f\left(x_{k}\right)^{\prime} d_{k}}{\left\|d_{k}\right\|}=0
$$

holds.
Then
(C1) $\left\{x_{k}\right\} \in \mathcal{L}\left(f\left(x_{0}\right)\right)$ and any subsequence of $\left\{x_{k}\right\}$ has an accumulation point.
(C2) $\left\{f\left(x_{k}\right)\right\}$ is monotonically decreasing and there exists $\bar{f}$ such that

$$
\lim _{k \rightarrow \infty} f\left(x_{k}\right)=\bar{f}
$$

(C3) $\left\{\nabla f\left(x_{k}\right)\right\}$ is such that

$$
\lim _{k \rightarrow \infty}\left\|\nabla f\left(x_{k}\right)\right\|=0
$$

(C4) Any accumulation point $\bar{x}$ of $\left\{x_{k}\right\}$ is such that $\nabla f(\bar{x})=0$.
Proof. Conditions (C1) and (C2) are a simple consequence of (H1) and (H3). Note now that (H2) implies

$$
\epsilon\left\|\nabla f\left(x_{k}\right)\right\| \leq \frac{\left|\nabla f\left(x_{k}\right)^{\prime} d_{k}\right|}{\left\|d_{k}\right\|},
$$

for all $k$. As a result, and by (H4),

$$
\lim _{k \rightarrow \infty} \epsilon\left\|\nabla f\left(x_{k}\right)\right\| \leq \lim _{k \rightarrow \infty} \frac{\left|\nabla f\left(x_{k}\right)^{\prime} d_{k}\right|}{\left\|d_{k}\right\|}=0
$$

hence (C3) holds. Finally, let $\bar{x}$ be an accumulation point of the sequence $\left\{x_{k}\right\}$, i.e. there is a subsequence that converges to $\bar{x}$. For such a subsequence, and by continuity of $f$, one has

$$
\lim _{k \rightarrow \infty} \nabla f\left(x_{k}\right)=\nabla f(\bar{x})
$$

and, by (C3),

$$
\nabla f(\bar{x})=0
$$

which proves (C4).

Remark. Theorem 4 does not guarantee the convergence of the sequence $\left\{x_{k}\right\}$ to a unique accumulation point. Obviously $\left\{x_{k}\right\}$ has a unique accumulation point if either $\Omega \cap \mathcal{L}\left(f\left(x_{0}\right)\right)$ contains only one point or $x, y \in \Omega \cap \mathcal{L}\left(f\left(x_{0}\right)\right)$, with $x \neq y$ implies $f(x) \neq f(y)$. Finally, if the set $\Omega \cap \mathcal{L}\left(f\left(x_{0}\right)\right)$ contains a finite number of points, a sufficient condition for the existence of a unique accumulation point is

$$
\lim _{k \rightarrow \infty}\left\|x_{k+1}-x_{k}\right\|=0
$$

Remark. The angle condition can be replaced by the following one. There exists $\eta>0$ and $q>0$, both independent from $k$, such that

$$
\nabla f\left(x_{k}\right)^{\prime} d_{k} \leq-\eta\left\|\nabla f\left(x_{k}\right)\right\|^{q}\left\|d_{k}\right\|
$$

$\diamond$
The result illustrated in Theorem 4 requires the fulfillment of the angle condition or of a similar one, i.e. of a condition involving $\nabla f$. In many algorithms that do not make use of the gradient it may be difficult to check the validity of the angle condition, hence it is necessary to use different conditions on the research directions. For example, it is possible to replace the angle condition with a property of linear independence of the research directions.

Theorem 5 Let $\left\{x_{k}\right\}$ be the sequence obtained by the algorithm

$$
x_{k+1}=x_{k}+\alpha_{k} d_{k},
$$

for $k \geq 0$. Assume that

- $\nabla^{2} f$ is continuous and the level set $\mathcal{L}\left(f\left(x_{0}\right)\right)$ is compact.
- There exist $\sigma>0$, independent from $k$, and $k_{0}>0$ such that, for any $k \geq k_{0}$ the matrix $P_{k}$ composed of the columns

$$
\frac{d_{k}}{\left\|d_{k}\right\|}, \frac{d_{k+1}}{\left\|d_{k+1}\right\|}, \ldots, \frac{d_{k+n-1}}{\left\|d_{k+n-1}\right\|},
$$

is such that

$$
\left|\operatorname{det} P_{k}\right| \geq \sigma
$$

- $\lim _{k \rightarrow \infty}\left\|x_{k+1}-x_{k}\right\|=0$.
- $f\left(x_{k+1}\right)<f\left(x_{k}\right)$ for any $k \geq 0$.
- The property

$$
\lim _{k \rightarrow \infty} \frac{\nabla f\left(x_{k}\right)^{\prime} d_{k}}{\left\|d_{k}\right\|}=0
$$

holds.

Then

- $\left\{x_{k}\right\} \in \mathcal{L}\left(f\left(x_{0}\right)\right)$ and any subsequence of $\left\{x_{k}\right\}$ has an accumulation point.
- $\left\{f\left(x_{k}\right)\right\}$ is monotonically decreasing and there exists $\bar{f}$ such that

$$
\lim _{k \rightarrow \infty} f\left(x_{k}\right)=\bar{f}
$$

- Any accumulation point $\bar{x}$ of $\left\{x_{k}\right\}$ is such that $\nabla f(\bar{x})=0$.

Moreover, if the set $\Omega \cap \mathcal{L}\left(f\left(x_{0}\right)\right)$ is composed of a finite number of points, the sequence $\left\{x_{k}\right\}$ has a unique accumulation point.

### 2.3.4 Speed of convergence

Together with the property of convergence of the sequence $\left\{x_{k}\right\}$ it is important to study also the speed of convergence. To study such a notion it is convenient to assume that $\left\{x_{k}\right\}$ converges to a point $x_{\star}$.
If there exists a finite $k$ such that $x_{k}=x_{\star}$ then we say that the sequence $\left\{x_{k}\right\}$ has finite convergence. Note that if $\left\{x_{k}\right\}$ is generated by an algorithm, there is a stopping condition that has to be satisfied at step $k$.
If $x_{k} \neq x_{\star}$ for any finite $k$, it is possible (and convenient) to study the asymptotic properties of $\left\{x_{k}\right\}$. One criterion to estimate the speed of convergence is based on the behavior of the error $\mathcal{E}_{k}=\left\|x_{k}-x_{\star}\right\|$, and in particular on the relation between $\mathcal{E}_{k+1}$ and $\mathcal{E}_{k}$.
We say that $\left\{x_{k}\right\}$ has speed of convergence of order $p$ if

$$
\lim _{k \rightarrow \infty}\left(\frac{\mathcal{E}_{k+1}}{\mathcal{E}_{k}^{p}}\right)=C_{p}
$$

with $p \geq 1$ and $0<C_{p}<\infty$. Note that if $\left\{x_{k}\right\}$ has speed of convergence of order $p$ then

$$
\lim _{k \rightarrow \infty}\left(\frac{\mathcal{E}_{k+1}}{\mathcal{E}_{k}^{q}}\right)=0
$$

if $1 \leq q<p$, and

$$
\lim _{k \rightarrow \infty}\left(\frac{\mathcal{E}_{k+1}}{\mathcal{E}_{k}^{q}}\right)=\infty
$$

if $q>p$. Moreover, from the definition of speed of convergence, it is easy to see that if $\left\{x_{k}\right\}$ has speed of convergence of order $p$ then, for any $\epsilon>0$ there exists $k_{0}$ such that

$$
\mathcal{E}_{k+1} \leq\left(C_{p}+\epsilon\right) \mathcal{E}_{k}^{p},
$$

for any $k>k_{0}$.
In the cases $p=1$ or $p=2$ the following terminology is often used. If $p=1$ and $0<C_{1} \leq 1$ the speed of convergence is linear; if $p=1$ and $C_{1}>1$ the speed of convergence is sublinear; if

$$
\lim _{k \rightarrow \infty}\left(\frac{\mathcal{E}_{k+1}}{\mathcal{E}_{k}}\right)=0
$$

the speed of convergence is superlinear, and finally if $p=2$ the speed of convergence is quadratic.
Of special interest in optimization is the case of superlinear convergence, as this is the kind of convergence that can be established for the efficient minimization algorithms. Note that if $x_{k}$ has superlinear convergence to $x_{\star}$ then

$$
\lim _{k \rightarrow \infty} \frac{\left\|x_{k+1}-x_{k}\right\|}{\left\|x_{k}-x_{\star}\right\|}=1 .
$$

Remark. In some cases it is not possible to establish the existence of the limit

$$
\lim _{k \rightarrow \infty}\left(\frac{\mathcal{E}_{k+1}}{\mathcal{E}_{k}^{q}}\right)
$$

In these cases an estimate of the speed of convergence is given by

$$
Q_{p}=\limsup _{k \rightarrow \infty}\left(\frac{\mathcal{E}_{k+1}}{\mathcal{E}_{k}^{q}}\right) .
$$

### 2.4 Line search

A line search is a method to compute the step $\alpha_{k}$ along a given direction $d_{k}$. The choice of $\alpha_{k}$ affects both the convergence and the speed of convergence of the algorithm. In any line search one considers the function of one variable $\phi: \mathbb{R} \rightarrow \mathbb{R}$ defined as

$$
\phi(\alpha)=f\left(x_{k}+\alpha d_{k}\right)-f\left(x_{k}\right) .
$$

The derivative of $\phi(\alpha)$ with respect to $\alpha$ is given by

$$
\dot{\phi}(\alpha)=\nabla f\left(x_{k}+\alpha d_{k}\right)^{\prime} d_{k}
$$

provided that $\nabla f$ is continuous. Note that $\nabla f\left(x_{k}+\alpha d_{k}\right)^{\prime} d_{k}$ describes the slope of the tangent to the function $\phi(\alpha)$, and in particular

$$
\dot{\phi}(0)=\nabla f\left(x_{k}\right)^{\prime} d_{k}
$$

coincides with the directional derivative of $f$ at $x_{k}$ along $d_{k}$.
From the general convergence results described, we conclude that the line search has to enforce the following conditions

$$
\begin{gathered}
f\left(x_{k+1}\right)<f\left(x_{k}\right) \\
\lim _{k \rightarrow \infty} \frac{\nabla f\left(x_{k}\right)^{\prime} d_{k}}{\left\|d_{k}\right\|}=0
\end{gathered}
$$

and, whenever possible, also the condition

$$
\lim _{k \rightarrow \infty}\left\|x_{k+1}-x_{k}\right\|=0
$$

To begin with, we assume that the directions $d_{k}$ are such that

$$
\nabla f\left(x_{k}\right)^{\prime} d_{k}<0
$$

for all $k$, i.e. $d_{k}$ is a descent direction, and that it is possible to compute, for any fixed $x$, both $f$ and $\nabla f$. Finally, we assume that the level set $\mathcal{L}\left(f\left(x_{0}\right)\right)$ is compact.

### 2.4.1 Exact line search

The exact line search consists in finding $\alpha_{k}$ such that

$$
\phi\left(\alpha_{k}\right)=f\left(x_{k}+\alpha_{k} d_{k}\right)-f\left(x_{k}\right) \leq f\left(x_{k}+\alpha d_{k}\right)-f\left(x_{k}\right)=\phi(\alpha)
$$

for any $\alpha \geq 0$. Note that, as $d_{k}$ is a descent direction and the set

$$
\left\{\alpha \in \mathbb{R}^{+}: \phi(\alpha) \leq \phi(0)\right\}
$$

is compact, because of compactness of $\mathcal{L}\left(f\left(x_{0}\right)\right)$, there exists an $\alpha_{k}$ that minimizes $\phi(\alpha)$. Moreover, for such $\alpha_{k}$ one has

$$
\dot{\phi}\left(\alpha_{k}\right)=\nabla f\left(x_{k}+\alpha_{k} d_{k}\right)^{\prime} d_{k}=0,
$$

i.e. if $\alpha_{k}$ minimizes $\phi(\alpha)$ the gradient of $f$ at $x_{k}+\alpha_{k} d_{k}$ is orthogonal to the direction $d_{k}$. From a geometrical point of view, if $\alpha_{k}$ minimizes $\phi(\alpha)$ then the level surface of $f$ through the point $x_{k}+\alpha_{k} d_{k}$ is tangent to the direction $d_{k}$ at such a point. (If there are several points of tangency, $\alpha_{k}$ is the one for which $f$ has the smallest value).
The search of $\alpha_{k}$ that minimizes $\phi(\alpha)$ is very expensive, especially if $f$ is not convex. Moreover, in general, the whole minimization algorithm does not gain any special advantage from the knowledge of such optimal $\alpha_{k}$. It is therefore more convenient to use approximate methods, i.e. methods which are computationally simple and which guarantee particular convergence properties. Such methods are aimed at finding an interval of acceptable values for $\alpha_{k}$ subject to the following two conditions

- $\alpha_{k}$ has to guarantee a sufficient reduction of $f$;
- $\alpha_{k}$ has to be sufficiently distant from 0 , i.e. $x_{k}+\alpha_{k} d_{k}$ has to be sufficiently away from $x_{k}$.


### 2.4.2 Armijo method

Armijo method was the first non-exact linear search method.
Let $a>0, \sigma \in(0,1)$ and $\gamma \in(0,1 / 2)$ be given and define the set of points

$$
A=\left\{\alpha \in R: \alpha=a \sigma^{j}, j=0,1, \ldots\right\} .
$$



Figure 2.3: Geometrical interpretation of Armijo method.

Armijo method consists in finding the largest $\alpha \in A$ such that

$$
\phi(\alpha)=f\left(x_{k}+\alpha d_{k}\right)-f\left(x_{k}\right) \leq \gamma \alpha \nabla f\left(x_{k}\right)^{\prime} d_{k}=\gamma \alpha \dot{\phi}(0) .
$$

Armijo method can be implemented using the following (conceptual) algorithm.
Step 1. Set $\alpha=a$.
Step 2. If

$$
f\left(x_{k}+\alpha d_{k}\right)-f\left(x_{k}\right) \leq \gamma \alpha \nabla f\left(x_{k}\right)^{\prime} d_{k}
$$

set $\alpha_{k}=\alpha$ and STOP. Else go to Step 3.
Step 3. Set $\alpha=\sigma \alpha$, and go to Step 2.
From a geometric point of view (see Figure 2.3) the condition in Step 2 requires that $\alpha_{k}$ is such that $\phi\left(\alpha_{k}\right)$ is below the straight line passing through the point $(0, \phi(0))$ and with slope $\gamma \dot{\phi}(0)$. Note that, as $\gamma \in(0,1 / 2)$ and $\dot{\phi}(0)<0$, such a straight line has a slope smaller than the slope of the tangent at the curve $\phi(\alpha)$ at the point $(0, \phi(0))$. For Armijo method it is possible to prove the following convergence result.

Theorem 6 Let $f: \mathbb{R}^{n} \rightarrow \mathbb{R}$ and assume $\nabla f$ is continuous and $\mathcal{L}\left(f\left(x_{0}\right)\right)$ is compact. Assume $\nabla f\left(x_{k}\right)^{\prime} d_{k}<0$ for all $k$ and there exist $C_{1}>0$ and $C_{2}>0$ such that

$$
C_{1} \geq\left\|d_{k}\right\| \geq C_{2}\left\|\nabla f\left(x_{k}\right)\right\|^{q},
$$

for some $q>0$ and for all $k$.
Then Armijo method yields in a finite number of iterations a value of $\alpha_{k}>0$ satisfying the condition in Step 2. Moreover, the sequence obtained setting $x_{k+1}=x_{k}+\alpha_{k} d_{k}$ is such that

$$
f\left(x_{k+1}\right)<f\left(x_{k}\right),
$$

for all $k$, and

$$
\lim _{k \rightarrow \infty} \frac{\nabla f\left(x_{k}\right)^{\prime} d_{k}}{\left\|d_{k}\right\|}=0
$$

Proof. We only prove that the method cannot loop indefinitely between Step 2 and Step 3. In fact, if this is the case, then the condition in Step 2 will never be satisfied, hence

$$
\frac{f\left(x_{k}+a \sigma^{j} d_{k}\right)-f\left(x_{k}\right)}{a \sigma^{j}}>\gamma \nabla f\left(x_{k}\right)^{\prime} d_{k} .
$$

Note now that $\sigma^{j} \rightarrow 0$ as $j \rightarrow \infty$, and the above inequality for $j \rightarrow \infty$ is

$$
\nabla f\left(x_{k}\right)^{\prime} d_{k}>\gamma \nabla f\left(x_{k}\right)^{\prime} d_{k},
$$

which is not possible since $\gamma \in(0,1 / 2)$ and $\nabla f\left(x_{k}\right)^{\prime} d_{k} \neq 0$.
$\triangleleft$

Remark. It is interesting to observe that in Theorem 6 it is not necessary to assume that $x_{k+1}=x_{k}+\alpha_{k} d_{k}$. It is enough that $x_{k+1}$ is such that

$$
f\left(x_{k+1}\right) \leq f\left(x_{k}+\alpha_{k} d_{k}\right)
$$

where $\alpha_{k}$ is generated using Armijo method. This implies that all acceptable values of $\alpha$ are those such that

$$
f\left(x_{k}+\alpha d_{k}\right) \leq f\left(x_{k}+\alpha_{k} d_{k}\right) .
$$

As a result, Theorem 6 can be used to prove also the convergence of an algorithm based on the exact line search.

### 2.4.3 Goldstein conditions

The main disadvantage of Armijo method is in the fact that, to find $\alpha_{k}$, all points in the set $A$, starting from the point $\alpha=a$, have to be tested till the condition in Step 2 is fulfilled. There are variations of the method that do not suffer from this disadvantage. A criterion similar to Armijo's, but that allows to find an acceptable $\alpha_{k}$ in one step, is based on the so-called Goldstein conditions.
Goldstein conditions state that given $\gamma_{1} \in(0,1)$ and $\gamma_{2} \in(0,1)$ such that $\gamma_{1}<\gamma_{2}, \alpha_{k}$ is any positive number such that

$$
f\left(x_{k}+\alpha_{k} d_{k}\right)-f\left(x_{k}\right) \leq \alpha_{k} \gamma_{1} \nabla f\left(x_{k}\right)^{\prime} d_{k}
$$

i.e. there is a sufficient reduction in $f$, and

$$
f\left(x_{k}+\alpha_{k} d_{k}\right)-f\left(x_{k}\right) \geq \alpha_{k} \gamma_{2} \nabla f\left(x_{k}\right)^{\prime} d_{k}
$$

i.e. there is a sufficient distance between $x_{k}$ and $x_{k+1}$.

From a geometric point of view (see Figure 2.4) this is equivalent to select $\alpha_{k}$ as any point such that the corresponding value of $f$ is included between two straight lines, of slope


Figure 2.4: Geometrical interpretation of Goldstein method.
$\gamma_{1} \nabla f\left(x_{k}\right)^{\prime} d_{k}$ and $\gamma_{2} \nabla f\left(x_{k}\right)^{\prime} d_{k}$, respectively, and passing through the point $(0, \phi(0))$. As $0<\gamma_{1}<\gamma_{2}<1$ it is obvious that there exists always an interval $I=[\underline{\alpha}, \bar{\alpha}]$ such that Goldstein conditions hold for any $\alpha \in I$.
Note that, a result similar to Theorem 6, can be also established if the sequence $\left\{x_{k}\right\}$ is generated using Goldstein conditions.
The main disadvantage of Armijo and Goldstein methods is in the fact that none of them impose conditions on the derivative of the function $\phi(\alpha)$ in the point $\alpha_{k}$, or what is the same on the value of $\nabla f\left(x_{k+1}\right)^{\prime} d_{k}$. Such extra conditions are sometimes useful in establishing convergence results for particular algorithms. However, for simplicity, we omit the discussion of these more general conditions (known as Wolfe conditions).

### 2.4.4 Line search without derivatives

It is possible to construct methods similar to Armijo's or Goldstein's also in the case that no information on the derivatives of the function $f$ is available.
Suppose, for simplicity, that $\left\|d_{k}\right\|=1$, for all $k$, and that the sequence $\left\{x_{k}\right\}$ is generated by

$$
x_{k+1}=x_{k}+\alpha_{k} d_{k} .
$$

If $\nabla f$ is not available it is not possible to decide a priori if the direction $d_{k}$ is a descent direction, hence it is necessary to consider also negative values of $\alpha$.
We now describe the simplest line search method that can be constructed with the considered hypothesis. This method is a modification of Armijo method and it is known as parabolic search.
Given $\lambda_{0}>0, \sigma \in(0,1 / 2), \gamma>0$ and $\rho \in(0,1)$. Compute $\alpha_{k}$ and $\lambda_{k}$ such that one of the following conditions hold.
Condition (i)

- $\lambda_{k}=\lambda_{k-1}$;
- $\alpha_{k}$ is the largest value in the set

$$
A=\left\{\alpha \in \mathbb{R}: \alpha= \pm \sigma^{j}, j=0,1, \ldots\right\}
$$

such that

$$
f\left(x_{k}+\alpha_{k} d_{k}\right) \leq f\left(x_{k}\right)-\gamma \alpha_{k}^{2},
$$

or, equivalently, $\phi\left(\alpha_{k}\right) \leq-\gamma \alpha_{k}^{2}$.
Condition (ii)

- $\alpha_{k}=0, \lambda_{k} \leq \rho \lambda_{k-1}$;
- $\min \left(f\left(x_{k}+\lambda_{k} d_{k}\right), f\left(x_{k}-\lambda_{k} d_{k}\right)\right) \geq f\left(x_{k}\right)-\gamma \lambda_{k}^{2}$.

At each step it is necessary to satisfy either Condition (i) or Condition (ii). Note that this is always possible for any $d_{k} \neq 0$. Condition (i) requires that $\alpha_{k}$ is the largest number in the set $A$ such that $f\left(x_{k}+\alpha_{k} d_{k}\right)$ is below the parabola $f\left(x_{k}\right)-\gamma \alpha^{2}$. If the function $\phi(\alpha)$ has a stationary point for $\alpha=0$ then there may be no $\alpha \in A$ such that Condition (i) holds. However, in this case it is possible to find $\lambda_{k}$ such that Condition (ii) holds. If Condition (ii) holds then $\alpha_{k}=0$, i.e. the point $x_{k}$ remains unchanged and the algorithms continues with a new direction $d_{k+1} \neq d_{k}$.
For the parabolic search algorithm it is possible to prove the following convergence result.
Theorem 7 Let $f: \mathbb{R}^{n} \rightarrow \mathbb{R}$ and assume $\nabla f$ is continuous and $\mathcal{L}\left(f\left(x_{0}\right)\right)$ is compact. If $\alpha_{k}$ is selected following the conditions of the parabolic search and if $x_{k+1}=x_{k}+\alpha_{k} d_{k}$, with $\left\|d_{k}\right\|=1$ then the sequence $\left\{x_{k}\right\}$ is such that

$$
f\left(x_{k+1}\right) \leq f\left(x_{k}\right)
$$

for all $k$,

$$
\lim _{k \rightarrow \infty} \nabla f\left(x_{k}\right)^{\prime} d_{k}=0
$$

and

$$
\lim _{k \rightarrow \infty}\left\|x_{k+1}-x_{k}\right\|=0
$$

Proof. (Sketch) Note that Condition (i) implies $f\left(x_{k+1}\right)<f\left(x_{k}\right)$, whereas Condition (ii) implies $f\left(x_{k+1}\right)=f\left(x_{k}\right)$. Note now that if Condition (ii) holds for all $k \geq \bar{k}$, then $\alpha_{k}=0$ for all $k \geq \bar{k}$, i.e. $\left\|x_{k+1}-x_{k}\right\|=0$. Moreover, as $\lambda_{k}$ is reduced at each step, necessarily $\nabla f\left(x_{\bar{k}}\right)^{\prime} \bar{d}=0$, where $\bar{d}$ is a limit of the sequence $\left\{d_{k}\right\}$.

### 2.4.5 Implementation of a line search algorithm

On the basis of the conditions described so far it is possible to construct algorithms that yield $\alpha_{k}$ in a finite number of steps. One such an algorithm can be described as follows. (For simplicity we assume that $\nabla f$ is known.)

- Initial data. $x_{k}, f\left(x_{k}\right), \nabla f\left(x_{k}\right), \underline{\alpha}$ and $\bar{\alpha}$.
- Initial guess for $\alpha$. A possibility is to select $\alpha$ as the point in which a parabola through $(0, \phi(0))$ with derivative $\dot{\phi}(0)$ for $\alpha=0$ takes a pre-specified minimum value $f_{\star}$. Initially, i.e. for $k=0, f_{\star}$ has to be selected by the designer. For $k>0$ it is possible to select $f_{\star}$ such that

$$
f\left(x_{k}\right)-f_{\star}=f\left(x_{k-1}\right)-f\left(x_{k}\right)
$$

The resulting $\alpha$ is

$$
\alpha_{\star}=-2 \frac{f\left(x_{k}\right)-f_{\star}}{\nabla f\left(x_{k}\right)^{\prime} d_{k}}
$$

In some algorithms it is convenient to select $\alpha \leq 1$, hence the initial guess for $\alpha$ will be $\min \left(1, \alpha_{\star}\right)$.

- Computation of $\alpha_{k}$. A value for $\alpha_{k}$ is computed using a line search method. If $\alpha_{k} \leq \underline{\alpha}$ the direction $d_{k}$ may not be a descent direction. If $\alpha_{k} \geq \bar{\alpha}$ the level set $\mathcal{L}\left(f\left(x_{k}\right)\right)$ may not be compact. If $\alpha_{k} \notin[\underline{\alpha}, \bar{\alpha}]$ the line search fails, and it is necessary to select a new research direction $d_{k}$. Otherwise the line search terminates and $x_{k+1}=x_{k}+\alpha_{k} d_{k}$.


### 2.5 The gradient method

The gradient method consists in selecting, as research direction, the direction of the antigradient at $x_{k}$, i.e.

$$
d_{k}=-\nabla f\left(x_{k}\right)
$$

for all $k$. This selection is justified noting that the direction ${ }^{7}$

$$
-\frac{\nabla f\left(x_{k}\right)}{\left\|\nabla f\left(x_{k}\right)\right\|_{E}}
$$

is the direction that minimizes the directional derivative, among all direction with unitary Euclidean norm. In fact, by Schwartz inequality, one has

$$
\left|\nabla f\left(x_{k}\right)^{\prime} d\right| \leq\|d\|_{E}\left\|\nabla f\left(x_{k}\right)\right\|_{E}
$$

and the equality sign holds if and only if $d=\lambda \nabla f\left(x_{k}\right)$, with $\lambda \in \mathbb{R}$. As a consequence, the problem

$$
\min _{\|d\|_{E}=1} \nabla f\left(x_{k}\right)^{\prime} d
$$

[^6]has the solution $d_{\star}=-\frac{\nabla f\left(x_{k}\right)}{\left\|\nabla f\left(x_{k}\right)\right\|_{E}}$. For this reason, the gradient method is sometimes called the method of the steepest descent. Note however that the (local) optimality of the direction $-\nabla f\left(x_{k}\right)$ depends upon the selection of the norm, and that with a proper selection of the norm, any descent direction can be regarded as the steepest descent.
The real interest in the direction $-\nabla f\left(x_{k}\right)$ rests on the fact that, if $\nabla f$ is continuous, then the former is a continuous descent direction, which is zero only if the gradient is zero, i.e. at a stationary point.
The gradient algorithm can be schematized has follows.
Step 0. Given $x_{0} \in \mathbb{R}^{n}$.
Step 1. Set $k=0$.
Step 2. Compute $\nabla f\left(x_{k}\right)$. If $\nabla f\left(x_{k}\right)=0$ STOP. Else set $d_{k}=-\nabla f\left(x_{k}\right)$.
Step 3. Compute a step $\alpha_{k}$ along the direction $d_{k}$ with any line search method such that
$$
f\left(x_{k}+\alpha_{k} d_{k}\right) \leq f\left(x_{k}\right)
$$
and
$$
\lim _{k \rightarrow \infty} \frac{\nabla f\left(x_{k}\right)^{\prime} d_{k}}{\left\|d_{k}\right\|}=0
$$

Step 4. Set $x_{k+1}=x_{k}+\alpha_{k} d_{k}, k=k+1$. Go to Step 2.
By the general results established in Theorem 4, we have the following fact regarding the convergence properties of the gradient method.

Theorem 8 Consider $f: \mathbb{R}^{n} \rightarrow \mathbb{R}$. Assume $\nabla f$ is continuous and the level set $\mathcal{L}\left(f\left(x_{0}\right)\right)$ is compact. Then any accumulation point of the sequence $\left\{x_{k}\right\}$ generated by the gradient algorithm is a stationary point of $f$.


Figure 2.5: The function $\sqrt{\xi} \frac{\xi-1}{\xi+1}$.

To estimate the speed of convergence of the method we can consider the behavior of the method in the minimization of a quadratic function, i.e. in the case

$$
f(x)=\frac{1}{2} x^{\prime} Q x+c^{\prime} x+d,
$$

with $Q=Q^{\prime}>0$. In such a case it is possible to obtain the following estimate

$$
\left\|x_{k+1}-x_{\star}\right\| \leq \sqrt{\frac{\lambda_{M}}{\lambda_{m}}} \frac{\sqrt{\frac{\lambda_{M}}{\lambda_{m}}-1}}{\sqrt{\frac{\lambda_{M}}{\lambda_{m}}+1}}\left\|x_{k}-x_{\star}\right\|,
$$

where $\lambda_{M} \geq \lambda_{m}>0$ are the maximum and minimum eigenvalue of $Q$, respectively. Note that the above estimate is exact for some initial points $x_{0}$. As a result, if $\lambda_{M} \neq \lambda_{m}$ the gradient algorithm has linear convergence, however, if $\lambda_{M} / \lambda_{m}$ is large the convergence can be very slow (see Figure 2.5).
Finally, if $\lambda_{M} / \lambda_{m}=1$ the gradient algorithm converges in one step. From a geometric point of view the ratio $\lambda_{M} / \lambda_{m}$ expresses the ratio between the lengths of the maximum and the minimum axes of the ellipsoids, that constitute the level surfaces of $f$. If this ratio is big there are points from which the gradient algorithm converges very slowly, see e.g. Figure 2.6.


Figure 2.6: Behavior of the gradient algorithm.

In the non-quadratic case, the performance of the gradient method are unacceptable, especially if the level surfaces of $f$ have high curvature.

### 2.6 Newton's method

Newton's method, with all its variations, is the most important method in unconstrained optimization. Let $f: \mathbb{R}^{n} \rightarrow \mathbb{R}$ be a given function and assume that $\nabla^{2} f$ is continuous. Newton's method for the minimization of $f$ can be derived assuming that, given $x_{k}$, the point $x_{k+1}$ is obtained minimizing a quadratic approximation of $f$. As $f$ is two times differentiable, it is possible to write

$$
f\left(x_{k}+s\right)=f\left(x_{k}\right)+\nabla f\left(x_{k}\right)^{\prime} s+\frac{1}{2} s^{\prime} \nabla^{2} f\left(x_{k}\right) s+\beta\left(x_{k}, s\right),
$$

in which

$$
\lim _{\|s\| \rightarrow 0} \frac{\beta\left(x_{k}, s\right)}{\|s\|^{2}}=0
$$

For $\|s\|$ sufficiently small, it is possible to approximate $f\left(x_{k}+s\right)$ with its quadratic approximation

$$
q(s)=f\left(x_{k}\right)+\nabla f\left(x_{k}\right)^{\prime} s+\frac{1}{2} s^{\prime} \nabla^{2} f\left(x_{k}\right) s
$$

If $\nabla^{2} f\left(x_{k}\right)>0$, the value of $s$ minimizing $q(s)$ can be obtained setting to zero the gradient of $q(s)$, i.e.

$$
\nabla q(s)=\nabla f\left(x_{k}\right)+\nabla^{2} f\left(x_{k}\right) s=0
$$

yielding

$$
s=-\left[\nabla^{2} f\left(x_{k}\right)\right]^{-1} \nabla f\left(x_{k}\right)
$$

The point $x_{k+1}$ is thus given by

$$
x_{k+1}=x_{k}-\left[\nabla^{2} f\left(x_{k}\right)\right]^{-1} \nabla f\left(x_{k}\right) .
$$

Finally, Newton's method can be described by the simple scheme.
Step 0. Given $x_{0} \in \mathbb{R}^{n}$.
Step 1. Set $k=0$.
Step 2. Compute

$$
s=-\left[\nabla^{2} f\left(x_{k}\right)\right]^{-1} \nabla f\left(x_{k}\right) .
$$

Step 3. Set $x_{k+1}=x_{k}+s, k=k+1$. Go to Step 2.
Remark. An equivalent way to introduce Newton's method for unconstrained optimization is to regard the method as an algorithm for the solution of the system of $n$ non-linear equations in $n$ unknowns given by

$$
\nabla f(x)=0
$$

For, consider, in general, a system of $n$ equations in $n$ unknown

$$
F(x)=0,
$$

with $x \in \mathbb{R}^{n}$ and $F: \mathbb{R}^{n} \rightarrow \mathbb{R}^{n}$. If the Jacobian matrix of $F$ exists and is continuous, then one can write

$$
F(x+s)=F(x)+\frac{\partial F}{\partial x}(x) s+\gamma(x, s)
$$

with

$$
\lim _{\|s\| \rightarrow 0} \frac{\gamma(x, s)}{\|s\|}=0
$$

Hence, given a point $x_{k}$ we can determine $x_{k+1}=x_{k}+s$ setting $s$ such that

$$
F\left(x_{k}\right)+\frac{\partial F}{\partial x}\left(x_{k}\right) s=0
$$

If $\frac{\partial F}{\partial x}\left(x_{k}\right)$ is invertible we have

$$
s=-\left[\frac{\partial F}{\partial x}\left(x_{k}\right)\right]^{-1} F\left(x_{k}\right),
$$

hence Newton's method for the solution of the system of equation $F(x)=0$ is

$$
\begin{equation*}
x_{k+1}=x_{k}-\left[\frac{\partial F}{\partial x}\left(x_{k}\right)\right]^{-1} F\left(x_{k}\right), \tag{2.3}
\end{equation*}
$$

with $k=0,1, \ldots$. Note that, if $F(x)=\nabla f$, then the above iteration coincides with Newton's method for the minimization of $f$.

To study the convergence properties of Newton's method we can consider the algorithm for the solution of a set of non-linear equations, summarized in equation (2.3). The following local convergence result, providing also an estimate of the speed of convergence, can be proved.

Theorem 9 Let $F: \mathbb{R}^{n} \rightarrow \mathbb{R}^{n}$ and assume that $F$ is continuously differentiable in an open set $\mathcal{D} \subset \mathbb{R}^{n}$. Suppose moreover that

- there exists $x_{\star} \in \mathcal{D}$ such that $F\left(x_{\star}\right)=0$;
- the Jacobian matrix $\frac{\partial F}{\partial x}\left(x_{\star}\right)$ is non-singular;
- there exists $L>0$ such that ${ }^{8}$

$$
\left\|\frac{\partial F}{\partial x}(z)-\frac{\partial F}{\partial x}(y)\right\| \leq L\|z-y\|,
$$

for all $z \in \mathcal{D}$ and $y \in \mathcal{D}$.
Then there exists and open set $\mathcal{B} \subset \mathcal{D}$ such that for any $x_{0} \in \mathcal{B}$ the sequence $\left\{x_{k}\right\}$ generated by equation (2.3) remains in $\mathcal{B}$ and converges to $x_{\star}$ with quadratic speed of convergence.

The result in Theorem 9 can be easily recast as a result for the convergence of Newton's method for unconstrained optimization. For, it is enough to note that all hypotheses on $F$ and $\frac{\partial F}{\partial x}$ translate into hypotheses on $\nabla f$ and $\nabla^{2} f$. Note however that the result is only local and does not allow to distinguish between local minima and local maxima. To construct an algorithm for which the sequence $\left\{x_{k}\right\}$ does not converge to maxima, and for which global convergence, i.e. convergence from points outside the set $\mathcal{B}$, holds, it is possible to modify Newton's method considering a line search along the direction $d_{k}=-\left[\nabla^{2} f\left(x_{k}\right)\right]^{-1} \nabla f\left(x_{k}\right)$. As a result, the modified Newton's algorithm

$$
\begin{equation*}
x_{k+1}=x_{k}-\alpha_{k}\left[\nabla^{2} f\left(x_{k}\right)\right]^{-1} \nabla f\left(x_{k}\right), \tag{2.4}
\end{equation*}
$$

[^7]in which $\alpha_{k}$ is computed using any line search algorithm, is obtained. If $\nabla^{2} f$ is uniformly positive definite, and this implies that the function $f$ is convex, the direction $d_{k}=-\left[\nabla^{2} f\left(x_{k}\right)\right]^{-1} \nabla f\left(x_{k}\right)$ is a descent direction satisfying the condition of angle. Hence, by Theorem 4, we can conclude the (global) convergence of the algorithm (2.4). Moreover, it is possible to prove that, for $k$ sufficiently large, the step $\alpha_{k}=1$ satisfies the conditions of Armijo method, hence the sequence $\left\{x_{k}\right\}$ has quadratic speed of convergence.
Remark. If the function to be minimized is quadratic, i.e.
$$
f(x)=\frac{1}{2} x^{\prime} Q x+c^{\prime} x+d,
$$
and if $Q>0$, Newton's method yields the (global) minimum of $f$ in one step. $\diamond$
In general, i.e. if $\nabla^{2} f(x)$ is not positive definite for all $x$, Newton's method may be inapplicable because either $\nabla^{2} f\left(x_{k}\right)$ is not invertible, or $d_{k}=-\left[\nabla^{2} f\left(x_{k}\right)\right]^{-1} \nabla f\left(x_{k}\right)$ is not a descent direction. In these cases it is necessary to further modify Newton's method. Diverse criteria have been proposed, most of which rely on the substitution of the matrix $\nabla^{2} f\left(x_{k}\right)$ with a matrix $M_{k}>0$ which is close in some sense to $\nabla^{2} f\left(x_{k}\right)$. A simpler modification can be obtained using the direction $d_{k}=-\nabla f\left(x_{k}\right)$ whenever the direction $d_{k}=-\left[\nabla^{2} f\left(x_{k}\right)\right]^{-1} \nabla f\left(x_{k}\right)$ is not a descent direction. This modification yields the following algorithm.

Step 0. Given $x_{0} \in \mathbb{R}^{n}$ and $\epsilon>0$.
Step 1. Set $k=0$.
Step 2. Compute $\nabla f\left(x_{k}\right)$. If $\nabla f\left(x_{k}\right)=0$ STOP. Else compute $\nabla^{2} f\left(x_{k}\right)$. If $\nabla^{2} f\left(x_{k}\right)$ is singular set $d_{k}=-\nabla f\left(x_{k}\right)$ and go to Step 6.
Step 3. Compute Newton direction $s$ solving the (linear) system

$$
\nabla^{2} f\left(x_{k}\right) s=-\nabla f\left(x_{k}\right)
$$

## Step 4. If

$$
\left|\nabla f\left(x_{k}\right)^{\prime} s\right|<\epsilon\left\|\nabla f\left(x_{k}\right)\right\|\|s\|
$$

set $d_{k}=-\nabla f\left(x_{k}\right)$ and go to Step 6.
Step 5. If

$$
\nabla f\left(x_{k}\right)^{\prime} s<0
$$

set $d_{k}=s$; if

$$
\nabla f\left(x_{k}\right)^{\prime} s>0
$$

set $d_{k}=-s$.
Step 6. Make a line search along $d_{k}$ assuming as initial estimate $\alpha=1$. Compute $x_{k+1}=x_{k}+\alpha_{k} d_{k}$, set $k=k+1$ and go to Step 2.

The above algorithm is such that the direction $d_{k}$ satisfies the condition of angle, i.e.

$$
\nabla f\left(x_{k}\right)^{\prime} d_{k} \leq-\epsilon\left\|\nabla f\left(x_{k}\right)\right\|\left\|d_{k}\right\|
$$

for all $k$. Hence, the convergence is guaranteed by the general result in Theorem 4. Moreover, if $\epsilon$ is sufficiently small, if the hypotheses of Theorem 9 hold, and if the line search is performed with Armijo method and with the initial guess $\alpha=1$, then the above algorithm has quadratic speed of convergence.
Finally, note that it is possible to modify Newton's method, whenever it is not applicable, without making use of the direction of the anti-gradient. We now briefly discuss two such modifications.

### 2.6.1 Method of the trust region

A possible approach to modify Newton's method to yield global convergence is to set the direction $d_{k}$ and the step $\alpha_{k}$ in such a way to minimize the quadratic approximation of $f$ on a sphere centered at $x_{k}$ and of radius $a_{k}$. Such a sphere is called trust region. This name refers to the fact that, in a small region around $x_{k}$ we are confident (we trust) that the quadratic approximation of $f$ is a good approximation.
The method of the trust region consists in selecting $x_{k+1}=x_{k}+s_{k}$, where $s_{k}$ is the solution of the problem

$$
\begin{equation*}
\min _{\|s\| \leq a_{k}} q(s) \tag{2.5}
\end{equation*}
$$

with

$$
q(s)=f\left(x_{k}\right)+\nabla f\left(x_{k}\right)^{\prime} s+\frac{1}{2} s^{\prime} \nabla^{2} f\left(x_{k}\right) s
$$

and $a_{k}>0$ the estimate at step $k$ of the trust region. As the above (constrained) optimization problem has always a solution, the direction $s_{k}$ is always defined. The computation of the estimate $a_{k}$ is done, iteratively, in such a way to enforce the condition $f\left(x_{k+1}\right)<f\left(x_{k}\right)$ and to make sure that $f\left(x_{k}+s_{k}\right) \approx q\left(s_{k}\right)$, i.e. that the change of $f$ and the estimated change of $f$ are close.
Using these simple ingredients it is possible to construct the following algorithm.
Step 0. Given $x_{0} \in \mathbb{R}^{n}$ and $a_{0}>0$.
Step 1. Set $k=0$.
Step 2. Compute $\nabla f\left(x_{k}\right)$. If $\nabla f\left(x_{k}\right)=0$ STOP. Else go to Step 3.
Step 3. Compute $s_{k}$ solving problem (2.5).
Step 4. Compute ${ }^{9}$

$$
\begin{equation*}
\rho_{k}=\frac{f\left(x_{k}+s_{k}\right)-f\left(x_{k}\right)}{q\left(s_{k}\right)-f\left(x_{k}\right)} \tag{2.6}
\end{equation*}
$$

[^8]Step 5. If $\rho_{k}<1 / 4$ set $a_{k+1}=\left\|s_{k}\right\| / 4$. If $\rho_{k}>3 / 4$ and $\left\|s_{k}\right\|=a_{k}$ set $a_{k+1}=2 a_{k}$. Else set $a_{k+1}=a_{k}$.

Step 6. If $\rho_{k} \leq 0$ set $x_{k+1}=x_{k}$. Else set $x_{k+1}=x_{k}+s_{k}$.
Step 7. Set $k=k+1$ and go to Step 2.
Remark. Equation (2.6) expresses the ratio between the actual change of $f$ and the estimated change of $f$.

It is possible to prove that, if $\mathcal{L}\left(f\left(x_{0}\right)\right)$ is compact and $\nabla^{2} f$ is continuous, any accumulation point resulting from the above algorithm is a stationary point of $f$, in which the second order necessary conditions hold.
The update of $a_{k}$ is devised to enlarge or shrink the region of confidence on the basis of the number $\rho_{k}$. It is possible to show that if $\left\{x_{k}\right\}$ converges to a local minimum in which $\nabla^{2} f$ is positive definite, then $\rho_{k}$ converges to one and the direction $s_{k}$ coincides, for $k$ sufficiently large, with the Newton direction. As a result, the method has quadratic speed of convergence.
In practice, the solution of the problem (2.5) cannot be obtained analytically, hence approximate problems have to be solved. For, consider $s_{k}$ as the solution of the equation

$$
\begin{equation*}
\left(\nabla^{2} f\left(x_{k}\right)+\nu_{k} I\right) s_{k}=-\nabla f\left(x_{k}\right), \tag{2.7}
\end{equation*}
$$

in which $\nu_{k}>0$ has to be determined with proper considerations. Under certain hypotheses, the $s_{k}$ determined solving equation (2.7) coincides with the $s_{k}$ computed using the method of the trust region.

Remark. A potential disadvantage of the method of the trust region is to reduce the step along Newton direction even if the selection $\alpha_{k}=1$ would be feasible.

### 2.6.2 Non-monotonic line search

Experimental evidence shows that Newton's method gives the best result if the step $\alpha_{k}=1$ is used. Therefore, the use of $\alpha_{k}<1$ along Newton direction, resulting e.g. from the application of Armijo method, results in a degradation of the performance of the algorithm. To avoid this phenomenon it has been suggested to relax the condition $f\left(x_{k+1}\right)<f\left(x_{k}\right)$ imposed on Newton algorithm, thus allowing the function $f$ to increase for a certain number of steps. For example, it is possible to substitute the reduction condition of Armijo method with the condition

$$
f\left(x_{k}+\alpha_{k} d_{k}\right) \leq \max _{0 \leq j \leq M}\left[f\left(x_{k-j}\right)\right]+\gamma \alpha_{k} \nabla f\left(x_{k}\right)^{\prime} d_{k}
$$

for all $k \geq M$, where $M>0$ is a fixed integer independent from $k$.

|  | Gradient method | Newton's method |
| :---: | :---: | :---: |
| Information required at <br> each step | $f$ and $\nabla f$ | $f, \nabla f$ and $\nabla^{2} f$ |
| Computation to find <br> the research direction | $\nabla f\left(x_{k}\right)$ | $\nabla f\left(x_{k}\right), \nabla^{2} f\left(x_{k}\right)$, <br> $-\left[\nabla^{2} f\left(x_{k}\right)\right]^{-1} \nabla f\left(x_{k}\right)$ |
| Convergence | Global if $\mathcal{L}\left(f\left(x_{0}\right)\right)$ <br> compact and $\nabla f$ <br> continuous | Local, but may be <br> rendered global |
| Behavior for quadratic <br> functions | Asymptotic <br> convergence | Convergence in one <br> step |
| Speed of convergence | Linear for quadratic <br> functions | Quadratic (under <br> proper hypotheses) |

Table 2.1: Comparison between the gradient method and Newton's method.

### 2.6.3 Comparison between Newton's method and the gradient method

The gradient method and Newton's method can be compared from different point of views, as described in Table 2.1. From the table, it is obvious that Newton's method has better convergence properties but it is computationally more expensive. There exist methods which preserve some of the advantages of Newton's method, namely speed of convergence faster than the speed of the gradient method and finite convergence for quadratic functions, without requiring the knowledge of $\nabla^{2} f$. Such methods are

- the conjugate directions methods;
- quasi-Newton methods.


### 2.7 Conjugate directions methods

Conjugate directions methods have been motivated by the need of improving the convergence speed of the gradient method, without requiring the computation of $\nabla^{2} f$, as required in Newton's method.
A basic characteristic of conjugate directions methods is to find the minimum of a quadratic function in a finite number of steps. These methods have been introduced for the solution of systems of linear equations and have later been extended to the solution of unconstrained optimization problems for non-quadratic functions.

Definition 5 Given a matrix $Q=Q^{\prime}$, the vectors $d_{1}$ and $d_{2}$ are said to be $Q$-conjugate if

$$
d_{1}^{\prime} Q d_{2}=0
$$

Remark. If $Q=I$ then two vectors are $Q$-conjugate if they are orthogonal.

Theorem 10 Let $Q \in \mathbb{R}^{n \times n}$ and $Q=Q^{\prime}>0$. Let $d_{i} \in \mathbb{R}^{n}$, for $i=0, \cdots, k$, be non-zero vectors. If $d_{i}$ are mutually $Q$-conjugate, i.e.

$$
d_{i}^{\prime} Q d_{j}=0
$$

for all $i \neq j$, then the vectors $d_{i}$ are linearly independent.
Proof. Suppose there exists constants $\alpha_{i}$, with $\alpha_{i} \neq 0$ for some $i$, such that

$$
\alpha_{0} d_{0}+\cdots \alpha_{k} d_{k}=0
$$

Then, left multiplying with $Q$ and $d_{j}^{\prime}$ yields

$$
\alpha_{j} d_{j}^{\prime} Q d_{j}=0
$$

which implies, as $Q>0, \alpha_{j}=0$. Repeating the same considerations for all $j \in[0, k]$ yields the claim.

Consider now a quadratic function

$$
f(x)=\frac{1}{2} x^{\prime} Q x+c^{\prime} x+d,
$$

with $x \in \mathbb{R}^{n}$ and $Q=Q^{\prime}>0$. The (global) minimum of $f$ is given by

$$
x_{\star}=-Q^{-1} c,
$$

and this can be computed using the procedure given in the next statement.
Theorem 11 Let $Q=Q^{\prime}>0$ and let $d_{0}, d_{1}, \cdots, d_{n-1}$ be $n$ non-zero vectors mutually $Q$-conjugate. Consider the algorithm

$$
x_{k+1}=x_{k}+\alpha_{k} d_{k}
$$

with

$$
\alpha_{k}=-\frac{\nabla f\left(x_{k}\right)^{\prime} d_{k}}{d_{k}^{\prime} Q d_{k}}=-\frac{\left(x_{k}^{\prime} Q+c^{\prime}\right) d_{k}}{d_{k}^{\prime} Q d_{k}} .
$$

Then, for any $x_{0}$, the sequence $\left\{x_{k}\right\}$ converges, in at most $n$ steps, to $x_{\star}=-Q^{-1} c$, i.e. it converges to the minimum of the quadratic function $f$.

Remark. Note that $\alpha_{k}$ is selected at each step to minimize the function $f\left(x_{k}+\alpha d_{k}\right)$ with respect to $\alpha$, i.e. at each step an exact line search in the direction $d_{k}$ is performed. $\diamond$

In the above statement we have assumed that the directions $d_{k}$ have been preliminarily assigned. However, it is possible to construct a procedure in which the directions are computed iteratively. For, consider the quadratic function $f(x)=\frac{1}{2} x^{\prime} Q x+c^{\prime} x+d$, with $Q>0$, and the following algorithm, known as conjugate gradient method.

Step 0. Given $x_{0} \in \mathbb{R}^{n}$ and the direction

$$
d_{0}=-\nabla f\left(x_{0}\right)=-\left(Q x_{0}+c\right)
$$

Step 1. Set $k=0$.
Step 2. Let

$$
x_{k+1}=x_{k}+\alpha_{k} d_{k}
$$

with

$$
\alpha_{k}=-\frac{\nabla f\left(x_{k}\right)^{\prime} d_{k}}{d_{k}^{\prime} Q d_{k}}-\frac{\left(x_{k}^{\prime} Q+c^{\prime}\right) d_{k}}{d_{k}^{\prime} Q d_{k}} .
$$

Step 3. Compute $d_{k+1}$ as follows

$$
d_{k+1}=-\nabla f\left(x_{k+1}\right)+\beta_{k} d_{k},
$$

with

$$
\beta_{k}=\frac{\nabla f\left(x_{k+1}\right)^{\prime} Q d_{k}}{d_{k}^{\prime} Q d_{k}}
$$

Step 4. Set $k=k+1$ and go to Step 2.

Remark. As already observed, $\alpha_{k}$ is selected to minimize the function $f\left(x_{k}+\alpha d_{k}\right)$. Moreover, this selection of $\alpha_{k}$ is also such that

$$
\begin{equation*}
\nabla f\left(x_{k+1}\right)^{\prime} d_{k}=0 . \tag{2.8}
\end{equation*}
$$

In fact,

$$
Q x_{k+1}=Q x_{k}+\alpha_{k} Q d_{k}
$$

hence

$$
\begin{equation*}
\nabla f\left(x_{k+1}\right)=\nabla f\left(x_{k}\right)+\alpha_{k} Q d_{k} . \tag{2.9}
\end{equation*}
$$

Left multiplying with $d_{k}^{\prime}$ yields

$$
d_{k}^{\prime} \nabla f\left(x_{k+1}\right)=d_{k}^{\prime} \nabla f\left(x_{k}\right)+d_{k}^{\prime} Q d_{k} \alpha_{k}=d_{k}^{\prime} \nabla f\left(x_{k}\right)-d_{k}^{\prime} Q d_{k} \frac{\nabla f\left(x_{k}\right)^{\prime} d_{k}}{d_{k}^{\prime} Q d_{k}}=0
$$

Remark. $\beta_{k}$ is such that $d_{k+1}$ is $Q$-conjugate with respect to $d_{k}$. In fact,

$$
d_{k}^{\prime} Q d_{k+1}=d_{k}^{\prime} Q\left(-\nabla f\left(x_{k+1}\right)+\frac{\nabla f\left(x_{k+1}\right)^{\prime} Q d_{k}}{d_{k}^{\prime} Q d_{k}} d_{k}\right)=d_{k}^{\prime} Q\left(-\nabla f\left(x_{k+1}\right)+\nabla f\left(x_{k+1}\right)\right)=0 .
$$

Moreover, this selection of $\beta_{k}$ yields also

$$
\begin{equation*}
\nabla f\left(x_{k}\right)^{\prime} d_{k}=-\nabla f\left(x_{k}\right)^{\prime} \nabla f\left(x_{k}\right) \tag{2.10}
\end{equation*}
$$

For the conjugate gradient method it is possible to prove the following fact.
Theorem 12 The conjugate gradient method yields the minimum of the quadratic function

$$
f(x)=\frac{1}{2} x^{\prime} Q x+c^{\prime} x+d
$$

with $Q=Q^{\prime}>0$, in at most $n$ iterations, i.e. there exists $m \leq n-1$ such that

$$
\nabla f\left(x_{m+1}\right)=0 .
$$

Moreover

$$
\begin{equation*}
\nabla f\left(x_{j}\right)^{\prime} \nabla f\left(x_{i}\right)=0 \tag{2.11}
\end{equation*}
$$

and

$$
\begin{equation*}
d_{j}^{\prime} Q d_{i}=0, \tag{2.12}
\end{equation*}
$$

for all $[0, m+1] \ni i \neq j \in[0, m+1]$.
Proof. To prove the (finite) convergence of the sequence $\left\{x_{k}\right\}$ it is enough to show that the directions $d_{k}$ are $Q$-conjugate, i.e. that equation (2.12) holds. In fact, if equation (2.12) holds the claim is a consequence of Theorem 11.

The conjugate gradient algorithm, in the form described above, cannot be used for the minimization of non-quadratic functions, as it requires the knowledge of the matrix $Q$, which is the Hessian of the function $f$. Note that the matrix $Q$ appears at two levels in the algorithm: in the computation of the scalar $\beta_{k}$ required to compute the new direction of research, and in the computation of the step $\alpha_{k}$. It is therefore necessary to modify the algorithm to avoid the computation of $\nabla^{2} f$, but at the same time it is reasonable to make sure that the modified algorithm coincides with the above one in the quadratic case.

### 2.7.1 Modification of $\beta_{k}$

To begin with note that, by equation (2.9), $\beta_{k}$ can be written as

$$
\beta_{k}=\frac{\nabla f\left(x_{k+1}\right)^{\prime} \frac{\nabla f\left(x_{k+1}\right)-\nabla f\left(x_{k}\right)}{\alpha_{k}}}{d_{k}^{\prime} \frac{\nabla f\left(x_{k+1}\right)-\nabla f\left(x_{k}\right)}{\alpha_{k}}}=\frac{\nabla f\left(x_{k+1}\right)^{\prime}\left[\nabla f\left(x_{k+1}\right)-\nabla f\left(x_{k}\right)\right]}{d_{k}^{\prime}\left[\nabla f\left(x_{k+1}\right)-\nabla f\left(x_{k}\right)\right]},
$$

and, by equation (2.8),

$$
\begin{equation*}
\beta_{k}=-\frac{\nabla f\left(x_{k+1}\right)^{\prime}\left[\nabla f\left(x_{k+1}\right)-\nabla f\left(x_{k}\right)\right]}{d_{k}^{\prime} \nabla f\left(x_{k}\right)} . \tag{2.13}
\end{equation*}
$$

Using equation (2.13), it is possible to construct several expressions for $\beta_{k}$, all equivalent in the quadratic case, but yielding different algorithms in the general (non-quadratic) case. A first possibility is to consider equations (2.10) and (2.11) and to define

$$
\begin{equation*}
\beta_{k}=\frac{\nabla f\left(x_{k+1}\right)^{\prime} \nabla f\left(x_{k+1}\right)}{\nabla f\left(x_{k}\right)^{\prime} \nabla f\left(x_{k}\right)}=\frac{\left\|\nabla f\left(x_{k+1}\right)\right\|^{2}}{\left\|\nabla f\left(x_{k}\right)\right\|^{2}}, \tag{2.14}
\end{equation*}
$$

which is known as Fletcher-Reeves formula.
A second possibility is to write the denominator as in equation (2.14) and the numerator as in equation (2.13), yielding

$$
\begin{equation*}
\beta_{k}=\frac{\nabla f\left(x_{k+1}\right)^{\prime}\left[\nabla f\left(x_{k+1}\right)-\nabla f\left(x_{k}\right)\right]}{\left\|\nabla f\left(x_{k}\right)\right\|^{2}}, \tag{2.15}
\end{equation*}
$$

which is known as Polak-Ribiere formula. Finally, it is possible to have the denominator as in (2.13) and the numerator as in (2.14), i.e.

$$
\begin{equation*}
\beta_{k}=-\frac{\left\|\nabla f\left(x_{k+1}\right)\right\|^{2}}{d_{k}^{\prime} \nabla f\left(x_{k}\right)} \tag{2.16}
\end{equation*}
$$

### 2.7.2 Modification of $\alpha_{k}$

As already observed, in the quadratic version of the conjugate gradient method also the step $\alpha_{k}$ depends upon $Q$. However, instead of using the $\alpha_{k}$ given in Step 2 of the algorithm, it is possible to use a line search along the direction $\alpha_{k}$. In this way, an algorithm for non-quadratic functions can be constructed. Note that $\alpha_{k}$, in the algorithm for quadratic functions, is also such that $d_{k} \nabla f\left(x_{k+1}\right)=0$. Therefore, in the line search, it is reasonable to select $\alpha_{k}$ such that, not only $f\left(x_{k+1}\right)<f\left(x_{k}\right)$, but also $d_{k}$ is approximately orthogonal to $\nabla f\left(x_{k+1}\right)$.
Remark. The condition of approximate orthogonality between $d_{k}$ and $\nabla f\left(x_{k+1}\right)$ cannot be enforced using Armijo method or Goldstein conditions. However, there are more sophisticated line search algorithms, known as Wolfe conditions, which allow to enforce the above constraint.

### 2.7.3 Polak-Ribiere algorithm

As a result of the modifications discussed in the last sections, it is possible to construct an algorithm for the minimization of general functions. For example, using equation (2.15) we obtain the following algorithm, due to Polak-Ribiere, which has proved to be one of the most efficient among the class of conjugate directions methods.

Step 0. Given $x_{0} \in \mathbb{R}^{n}$.
Step 1. Set $k=0$.
Step 2. Compute $\nabla f\left(x_{k}\right)$. If $\nabla f\left(x_{k}\right)=0$ STOP. Else let

$$
d_{k}= \begin{cases}-\nabla f\left(x_{0}\right), & \text { if } k=0 \\ -\nabla f\left(x_{k}\right)+\frac{\nabla f\left(x_{k}\right)^{\prime}\left[\nabla f\left(x_{k}\right)-\nabla f\left(x_{k-1}\right)\right]}{\left\|\nabla f\left(x_{k-1}\right)\right\|^{2}} d_{k-1}, & \text { if } k \geq 1\end{cases}
$$

Step 3. Compute $\alpha_{k}$ performing a line search along $d_{k}$.
Step 4. Set $x_{k+1}=x_{k}+\alpha_{k} d_{k}, k=k+1$ and go to Step 2.
Remark. The line search has to be sufficiently accurate, to make sure that all directions generated by the algorithm are descent directions. A suitable line search algorithm is the so-called Wolfe method, which is a modification of Goldstein method.

Remark. To guarantee global convergence of a subsequence it is possible to use, every $n$ steps, the direction $-\nabla f$. In this case, it is said that the algorithm uses a restart procedure. For the algorithm with restart it is possible to have quadratic speed of convergence in $n$ steps, i.e

$$
\left\|x_{k+n}-x_{\star}\right\| \leq \gamma\left\|x_{k}-x_{\star}\right\|^{2}
$$

for some $\gamma>0$.

Remark. It is possible to modify Polak-Ribiere algorithm to make sure that at each step the angle condition holds. In this case, whenever the direction $d_{k}$ does not satisfy the angle condition, it is sufficient to use the direction $-\nabla f$. Note that, enforcing the angle condition, yields a globally convergent algorithm.

Remark. Even if the use of the direction $-\nabla f$ every $n$ steps, or whenever the angle condition is not satisfied, allows to prove global convergence of Polak-Ribiere algorithm, it has been observed in numerical experiments that such modified algorithms do not perform as well as the original one.

### 2.8 Quasi-Newton methods

Conjugate gradient methods have proved to be more efficient than the gradient method. However, in general, it is not possible to guarantee superlinear convergence. The main advantage of conjugate gradient methods is in the fact that they do not require to construct and store any matrix, hence can be used in large scale problems.

In small and medium scale problems, i.e. problems with less then a few hundreds decision variables, in which $\nabla^{2} f$ is not available, it is convenient to use the so-called quasi-Newton methods.
Quasi Newton methods, as conjugate directions methods, have been introduced for quadratic functions. They are described by an algorithm of the form

$$
x_{k+1}=x_{k}-\alpha_{k} H_{k} \nabla f\left(x_{k}\right)
$$

with $H_{0}$ given. The matrix $H_{k}$ is an approximation of $\left[\nabla^{2} f\left(x_{k}\right)\right]^{-1}$ and it is computed iteratively at each step.
If $f$ is a quadratic function, the gradient of $f$ is given by

$$
\nabla f(x)=Q x+c
$$

for some $Q$ and $c$, hence for any $x \in \mathbb{R}^{n}$ and $y \in \mathbb{R}^{n}$ one has

$$
\nabla f(y)-\nabla f(x)=Q(y-x)
$$

or, equivalently,

$$
Q^{-1}[\nabla f(y)-\nabla f(x)]=y-x
$$

It is then natural, in general, to construct the sequence $\left\{H_{k}\right\}$ such that

$$
\begin{equation*}
H_{k+1}\left[\nabla f\left(x_{k+1}\right)-\nabla f\left(x_{k}\right)\right]=x_{k+1}-x_{k} \tag{2.17}
\end{equation*}
$$

Equation (2.17) is known as quasi-Newton equation.
There exist several update methods satisfying the quasi-Newton equation. For simplicity, set

$$
\gamma_{k}=\nabla f\left(x_{k+1}\right)-\nabla f\left(x_{k}\right)
$$

and

$$
\delta_{k}=x_{k+1}-x_{k}
$$

As a result, equation (2.17) can be rewritten as

$$
H_{k+1} \gamma_{k}=\delta_{k}
$$

One of the first quasi-Newton methods has been proposed by Davidon, Fletcher and Powell, and can be summarized by the equations

$$
\operatorname{DFP}\left\{\begin{align*}
H_{0} & =I  \tag{2.18}\\
H_{k+1} & =H_{k}+\frac{\delta_{k} \delta_{k}^{\prime}}{\delta_{k}^{\prime} \gamma_{k}}-\frac{H_{k} \gamma_{k} \gamma_{k}^{\prime} H_{k}}{\gamma_{k}^{\prime} H_{k} \gamma_{k}}
\end{align*}\right.
$$

It is easy to show that the matrix $H_{k+1}$ satisfies the quasi-Newton equation (2.17), i.e.

$$
\begin{aligned}
H_{k+1} \gamma_{k} & =H_{k} \gamma_{k}+\frac{\delta_{k} \delta_{k}^{\prime}}{\delta_{k}^{\prime} \gamma_{k}} \gamma_{k}-\frac{H_{k} \gamma_{k} \gamma_{k}^{\prime} H_{k}}{\gamma_{k}^{\prime} H_{k} \gamma_{k}} \gamma_{k} \\
& =H_{k} \gamma_{k}+\frac{\delta_{k}^{\prime} \gamma_{k}}{\delta_{k}^{\prime} \gamma_{k}} \delta_{k}-\frac{\gamma_{k}^{\prime} H_{k} \gamma_{k}}{\gamma_{k}^{\prime} H_{k} \gamma_{k}} H_{k} \gamma_{k} \\
& =\delta_{k}
\end{aligned}
$$

Moreover, it is possible to prove the following fact, which gives conditions such that the matrices generated by DFP method are positive definite for all $k$.

Theorem 13 Let $H_{k}=H_{k}^{\prime}>0$ and assume $\delta_{k}^{\prime} \gamma_{k}>0$. Then the matrix

$$
H_{k}+\frac{\delta_{k} \delta_{k}^{\prime}}{\delta_{k}^{\prime} \gamma_{k}}-\frac{H_{k} \gamma_{k} \gamma_{k}^{\prime} H_{k}}{\gamma_{k}^{\prime} H_{k} \gamma_{k}}
$$

is positive definite.
DFP method has the following properties. In the quadratic case, if $\alpha_{k}$ is selected to minimize

$$
f\left(x_{k}-\alpha H_{k} \nabla f\left(x_{k}\right)\right),
$$

then

- the directions $d_{k}=-H_{k} \nabla f\left(x_{k}\right)$ are mutually conjugate;
- the minimum of the (quadratic) function is found in at most $n$ steps, moreover $H_{n}=Q^{-1}$;
- the matrices $H_{k}$ are always positive definite.

In the non-quadratic case

- the matrices $H_{k}$ are positive definite (hence $d_{k}=-H_{k} \nabla f\left(x_{k}\right)$ is a descent direction) if $\delta_{k}^{\prime} \gamma_{k}>0$;
- it is globally convergent if $f$ is strictly convex and if the line search is exact;
- it has superlinear speed of convergence (under proper hypotheses).

A second, and more general, class of update formulae, including as a particular case DFP formula, is the so-called Broyden class, defined by the equations

$$
\text { Broyden }\left\{\begin{align*}
H_{0} & =I  \tag{2.19}\\
H_{k+1} & =H_{k}+\frac{\delta_{k} \delta_{k}^{\prime}}{\delta_{k}^{\prime} \gamma_{k}}-\frac{H_{k} \gamma_{k} \gamma_{k}^{\prime} H_{k}}{\gamma_{k}^{\prime} H_{k} \gamma_{k}}+\phi v_{k} v_{k}^{\prime}
\end{align*}\right.
$$

with $\phi \geq 0$ and

$$
v_{k}=\left(\gamma_{k}^{\prime} H_{k} \gamma_{k}\right)^{1 / 2}\left(\frac{\delta_{k}}{\delta_{k}^{\prime} \gamma_{k}}-\frac{H_{k} \gamma_{k}}{\gamma_{k}^{\prime} H_{k} \gamma_{k}}\right) .
$$

If $\phi=0$ then we obtain DFP formula, whereas for $\phi=1$ we have the so-called Broyden-Fletcher-Goldfarb-Shanno (BFGS) formula, which is one of the preferred algorithms in applications. From Theorem 13 it is easy to infer that, if $H_{0}>0, \gamma_{k}^{\prime} \delta_{k}>0$ and $\phi \geq 0$, then all formulae in the class of Broyden generate matrices $H_{k}>0$.

Remark. Note that the condition $\delta_{k}^{\prime} \gamma_{k}>0$ is equivalent to

$$
\left(\nabla f\left(x_{k+1}\right)-\nabla f\left(x_{k}\right)\right)^{\prime} d_{k}>0
$$

and this can be enforced with a sufficiently precise line search.
$\diamond$

For the method based on BFGS formula, a global convergence result, for convex functions and in the case of non-exact (but sufficiently accurate) line search, has been proved. Moreover, it has been shown that the algorithm has superlinear speed of convergence. This algorithm can be summarized as follows.

Step 0. Given $x_{0} \in \mathbb{R}^{n}$.
Step 1. Set $k=0$.
Step 2. Compute $\nabla f\left(x_{k}\right)$. If $\nabla f\left(x_{k}\right)=0$ STOP. Else compute $H_{k}$ with BFGS equation and set

$$
d_{k}=-H_{k} \nabla f\left(x_{k}\right)
$$

Step 3. Compute $\alpha_{k}$ performing a line search along $d_{k}$.
Step 4. Set $x_{k+1}=x_{k}+\alpha_{k} d_{k}, k=k+1$ and go to Step 2.
In the general case it is not possible to prove global convergence of the algorithm. However, this can be enforced verifying (at the end of Step 2), if the direction $d_{k}$ satisfies an angle condition, and if not use the direction $d_{k}=-\nabla f\left(x_{k}\right)$. However, as already observed, this modification improves the convergence properties, but reduces (sometimes drastically) the speed of convergence.

### 2.9 Methods without derivatives

All the algorithms that have been discussed presuppose the knowledge of the derivatives (first and/or second) of the function $f$. There are, however, also methods which do not require such a knowledge. These methods can be divided in two classes: direct research methods and methods using finite difference approximations.
Direct search methods are based upon the direct comparison of the values of the function $f$ in the points generated by the algorithm, without making use of the necessary condition of optimality $\nabla f=0$. In this class, the most interesting methods, i.e. the methods for which it is possible to give theoretical results, are those that make use cyclically of $n$ linearly independent directions. The simplest possible method, known as the method of the coordinate directions, can be described by the following algorithm.

Step 0. Given $x_{0} \in \mathbb{R}^{n}$.
Step 1. Set $k=0$.
Step 2. Set $j=1$.

Step 3. Set $d_{k}=e_{j}$, where $e_{j}$ is the $j$-th coordinate direction.
Step 4. Compute $\alpha_{k}$ performing a line search without derivatives along $d_{k}$.
Step 5. Set $x_{k+1}=x_{k}+\alpha_{k} d_{k}, k=k+1$.
Step 6. If $j<n$ set $j=j+1$ and go to Step 3. If $j=n$ go to Step 2.
It is easy to verify that the matrix

$$
P_{k}=\left[\begin{array}{llll}
d_{k} & d_{k+1} & \cdots & d_{k+n-1}
\end{array}\right]
$$

is such that

$$
\left|\operatorname{det} P_{k}\right|=1,
$$

hence, if the line search is such that

$$
\lim _{k \rightarrow \infty} \frac{\nabla f\left(x_{k}\right)^{\prime} d_{k}}{\left\|d_{k}\right\|}=0
$$

and

$$
\lim _{k \rightarrow \infty}\left\|x_{k+1}-x_{k}\right\|=0
$$

convergence to stationary points is ensured by the general result in Theorem 5. Note that, the line search can be performed using the parabolic line search method described in Section 2.4.4.
The method of the coordinate directions is not very efficient, in terms of speed of convergence. Therefore, a series of heuristics have been proposed to improve its performance. One such heuristics is the so-called method of Jeeves and Hooke, in which not only the search along the coordinate directions is performed, but also a search along directions joining pairs of points generated by the algorithm. In this way, the search is performed along what may be considered to be the most promising directions.
An alternative direct search method is the so-called simplex method (which should not be confused with the simplex method of linear programming). The method starts with $n+1$ (equally spaced) points $x_{(i)} \in \mathbb{R}^{n}$ (these points give a simplex in $\mathbb{R}^{n}$ ). In each of these points the function $f$ is computed and the vertex where the function $f$ attains the maximum value is determined. Suppose this is the vertex $x_{(n+1)}$. This vertex is reflected with respect to the center of the simplex, i.e. the point

$$
x_{c}=\frac{1}{n+1} \sum_{i=1}^{n+1} x_{(i)}
$$

As a result, the new vertex

$$
x_{(n+2)}=x_{c}+\alpha\left(x_{c}-x_{(n+1)}\right)
$$

where $\alpha>0$, is constructed, see Figure 2.7. The procedure is then repeated.


Figure 2.7: The simplex method. The points $x_{(1)}, x_{(2)}$ and $x_{(3)}$ yields the starting simplex. The second simplex is given by the points $x_{(1)}, x_{(2)}$ and $x_{(4)}$. The third simplex is given by the points $x_{(2)}, x_{(4)}$ and $x_{(5)}$.

It is possible that the vertex that is generated by one step of the algorithm is (again) the one where the function $f$ has its maximum. In this case, the algorithm cycles, hence the next vertex has to be determined using a different strategy. For example, it is possible to construct the next vertex by reflecting another of the remaining $n$ vertex, or to shrink the simplex.
As a stopping criterion it is possible to consider the condition

$$
\begin{equation*}
\frac{1}{n+1} \sum_{i=1}^{n+1}\left(f\left(x_{(i)}\right)-\bar{f}\right)^{2}<\epsilon \tag{2.20}
\end{equation*}
$$

where $\epsilon>0$ is assigned by the designer, and

$$
\bar{f}=\frac{1}{n+1} \sum_{i=1}^{n+1} f\left(x_{(i)}\right)
$$

i.e. $\bar{f}$ is the mean value of the $f\left(x_{(i)}\right)$. Condition (2.20) implies that the points $x_{(i)}$ are all in a region where the function $f$ is flat.
As already observed, direct search methods are not very efficient, and can be used only for problems with a few decision variables and when approximate solutions are acceptable. As an alternative, if the derivatives of the function $f$ are not available, it is possible to resort to numeric differentiation, e.g the entries of the gradient of $f$ can be computed using the so-called forward difference approximation, i.e.

$$
\frac{\partial f(x)}{\partial x_{i}} \approx \frac{f\left(x+t e_{i}\right)-f(x)}{t}
$$

where $e_{i}$ is the $i$-th column of the identity matrix of dimension $n$, and $t>0$ has to be fixed by the user. Note that there are methods for the computation of the optimal value of $t$, i.e. the value of $t$ which minimizes the approximation error.

Chapter 3
Nonlinear programming

### 3.1 Introduction

In this chapter we discuss the basic tools for the solution of optimization problems of the form

$$
P_{0}\left\{\begin{array}{l}
\min _{x} f(x)  \tag{3.1}\\
g(x)=0 \\
h(x) \leq 0
\end{array}\right.
$$

In the problem $P_{0}$ there are both equality and inequality constraints ${ }^{1}$. However, sometimes for simplicity, or because a method has been developed for problems with special structure, we will refer to problems with only equality constraints, i.e. to problems of the form

$$
P_{1}\left\{\begin{array}{l}
\min _{x} f(x)  \tag{3.2}\\
g(x)=0
\end{array}\right.
$$

or to problems with only inequality constraints, i.e. to problems of the form

$$
P_{2}\left\{\begin{array}{l}
\min _{x} f(x)  \tag{3.3}\\
h(x) \leq 0 .
\end{array}\right.
$$

In all the above problems we have $x \in \mathbb{R}^{n}, f: \mathbb{R}^{n} \rightarrow \mathbb{R}, g: \mathbb{R}^{n} \rightarrow \mathbb{R}^{m}$, and $h: \mathbb{R}^{n} \rightarrow \mathbb{R}^{p}$. From a formal point of view it is always possible to transform the equality constraint $g_{i}(x)=0$ into a pair of inequality constraints, i.e. $g_{i}(x) \leq 0$ and $-g_{i}(x) \leq 0$. Hence, problem $P_{1}$ can be (equivalently) described by

$$
\tilde{P}_{1}\left\{\begin{array}{l}
\min _{x} f(x) \\
g(x) \leq 0 \\
-g(x) \leq 0
\end{array}\right.
$$

which is a special case of problem $P_{2}$. In the same way, it is possible to transform the inequality constraint $h_{i}(x) \leq 0$ into the equality constraint $h_{i}(x)+y_{i}^{2}=0$, where $y_{i}$ is an auxiliary variable (also called slack variable). Therefore, defining the extended vector $z=\left[x^{\prime}, y^{\prime}\right]^{\prime}$, problem $P_{2}$ can be rewritten as

$$
\tilde{P}_{2}\left\{\begin{array}{l}
\min _{z} f(x) \\
h(x)+Y=0,
\end{array}\right.
$$

with

$$
Y=\left[\begin{array}{c}
y_{1}^{2} \\
y_{2}^{2} \\
\vdots \\
y_{p}^{2}
\end{array}\right]
$$

which is a special case of problem $P_{1}$.

[^9]Note however, that the transformation of equality constraints into inequality constraints yields an increase in the number of constraints, whereas the transformation of inequality constraints into equality constraints results in an increased number of variables.
Given problem $P_{0}$ (or $P_{1}$, or $P_{2}$ ), a point $x$ satisfying the constraints is said to be an admissible point, and the set of all admissible points is called the admissible set and it is denoted with $\mathcal{X}$. Note that the problem makes sense only if $\mathcal{X} \neq \emptyset$.
In what follows it is assumed that the functions $f, g$ and $h$ are two times differentiable, however we do not make any special hypothesis on the form of such functions. Note however, that if $g$ and $h$ are linear there are special algorithms, and linear/quadratic programming algorithms are used if $f$ is linear/quadratic and $g$ and $h$ are linear. We do not discuss these special algorithms, and concentrate mainly on algorithms suitable for general problems.

### 3.2 Definitions and existence conditions

Consider the problem $P_{0}$ (or $P_{1}$, or $P_{2}$ ). The following definitions are instrumental to provide a necessary condition and a sufficient condition for the existence of local minima.

Definition 6 An open ball with center $x^{\star}$ and radius $\theta>0$ is the set

$$
B\left(x^{\star}, \theta\right)=\left\{x \in \mathbb{R}^{n} \mid\left\|x-x^{\star}\right\|<\theta\right\} .
$$

Definition $7 A$ point $x^{\star} \in \mathcal{X}$ is a constrained local minimum if there exists $\theta>0$ such that

$$
\begin{equation*}
f(y) \geq f\left(x^{\star}\right), \tag{3.4}
\end{equation*}
$$

for all $y \in \mathcal{X} \cap B\left(x^{\star}, \theta\right)$.
A point $x^{\star} \in \mathcal{X}$ is a constrained global minimum if

$$
\begin{equation*}
f(y) \geq f\left(x^{\star}\right), \tag{3.5}
\end{equation*}
$$

for all $y \in \mathcal{X}$.
If the inequality (3.4) (or (3.5)) holds with a strict inequality sign for all $y \neq x^{\star}$ then the minimum is said to be strict.

Definition 8 The $i$-th inequality constraints $h_{i}(x)$ is said to be active at $\tilde{x}$ if $h_{i}(\tilde{x})=0$. The set $I_{a}(\tilde{x})$ is the set of all indexes $i$ such that $h_{i}(\tilde{x})=0$, i.e.

$$
I_{a}(\tilde{x})=\left\{i \in\{1,2, \cdots, p\} \mid h_{i}(\tilde{x})=0\right\} .
$$

The vector $h_{a}(\tilde{x})$ is the subvector of $h(x)$ corresponding to the active constraints, i.e.

$$
h_{a}(\tilde{x})=\left\{h_{i}(\tilde{x}) \mid i \in I_{a}(\tilde{x}) .\right.
$$

Definition 9 A point $\tilde{x}$ is a regular point for the constraints if at $\tilde{x}$ the gradients of the active constraints, i.e. the vectors $\nabla g_{i}(\tilde{x})$, for $i=1, \cdots, m$ and $\nabla h_{i}(\tilde{x})$, for $i \in I_{a}(\tilde{x})$, are linearly independent.

The definition of regular point is given because, the necessary and the sufficient conditions for optimality, in the case of regular points are relatively simple. To state these conditions, and with reference to problem $P_{0}$, consider the Lagrangian function

$$
\begin{equation*}
L(x, \lambda, \rho)=f(x)+\lambda^{\prime} g(x)+\rho^{\prime} h(x) \tag{3.6}
\end{equation*}
$$

with $\lambda \in \mathbb{R}^{m}$ and $\rho \in \mathbb{R}^{p}$. The vectors $\lambda$ and $\rho$ are called multipliers.
With the above ingredients and definitions it is now possible to provide a necessary condition and a sufficient condition for local optimality.

Theorem 14 [First order necessary condition] Consider problem $P_{0}$. Suppose $x^{\star}$ is a local solution of the problem $P_{0}$, and $x^{\star}$ is a regular point for the constraints.
Then there exist (unique) multipliers $\lambda^{\star}$ and $\rho^{\star}$ such that ${ }^{2}$

$$
\begin{align*}
& \nabla_{x} L\left(x^{\star}, \lambda^{\star}, \rho^{\star}\right)=0 \\
& g\left(x^{\star}\right)=0 \\
& h\left(x^{\star}\right) \leq 0  \tag{3.7}\\
& \rho^{\star} \geq 0 \\
& \left(\rho^{\star}\right)^{\prime} h\left(x^{\star}\right)=0 .
\end{align*}
$$

Conditions (3.7) are known as Kuhn-Tucker conditions.
Definition 10 Let $x^{\star}$ be a local solution of problem $P_{0}$ and let $\rho^{\star}$ be the corresponding (optimal) multiplier. At $x^{\star}$ the condition of strict complementarity holds if $\rho_{i}^{\star}>0$ for all $i \in I_{a}\left(x^{\star}\right)$.

Theorem 15 [Second order sufficient condition] Consider the problem $P_{0}$. Assume that there exist $x^{\star}, \lambda^{\star}$ and $\rho^{\star}$ satisfying conditions (3.7). Suppose moreover that $\rho^{\star}$ is such that the condition of strict complementarity holds at $x^{\star}$. Suppose finally that

$$
\begin{equation*}
s^{\prime} \nabla_{x x}^{2} L\left(x^{\star}, \lambda^{\star}, \rho^{\star}\right) s>0 \tag{3.8}
\end{equation*}
$$

for all $s \neq 0$ such that

$$
\left[\begin{array}{c}
\frac{\partial g\left(x^{\star}\right)}{\partial x} \\
\frac{\partial h_{a}\left(x^{\star}\right)}{\partial x}
\end{array}\right] s=0 .
$$

Then $x^{\star}$ is a strict constrained local minimum of problem $P_{0}$.
Remark. Necessary and sufficient conditions for a global minimum can be given under proper convexity hypotheses, i.e. if the function $f$ is convex in $\mathcal{X}$, and if $\mathcal{X}$ is a convex set. This is the case, for example if there are no inequality constraints and if the equality constraints are linear.

[^10]Remark. If all points in $\mathcal{X}$ are regular points for the constraints then conditions (3.7) yield a set of points $\mathcal{P}$, i.e. the points satisfying conditions (3.7), and among these points there are all constrained local minima (and also the constrained global minimum, if it exists). However, if there are points in $\mathcal{X}$ which are not regular points for the constraints, then the set $\mathcal{P}$ may not contain all constrained local minima. These have to be searched in the set $\mathcal{P}$ and in the set of non-regular points.

Remark. In what follows, we will always tacitly assume that the conditions of regularity and of strict complementarity hold.

### 3.2.1 A simple proof of Kuhn-Tucker conditions for equality constraints

Consider problem $P_{1}$, i.e. a minimization problem with only equality constraints, and a point $x^{\star}$ such that $g\left(x^{\star}\right)=0$, i.e. $x^{\star} \in \mathcal{X}$. Suppose that ${ }^{3}$

$$
\operatorname{rank} \frac{\partial g}{\partial x}\left(x^{\star}\right)=m
$$

i.e. $x^{\star}$ is a regular point for the constraints, and that $x^{\star}$ is a constrained local minimum. By the implicit function theorem, there exist a neighborhood of $x^{\star}$, a partition of the vector $x$, i.e.

$$
x=\left[\begin{array}{l}
u \\
v
\end{array}\right],
$$

with $u \in \mathbb{R}^{m}$ and $v \in \mathbb{R}^{n-m}$, and a function $\phi$ such that the constrains $g(x)=0$ can be (locally) rewritten as

$$
u=\phi(v) .
$$

As a result (locally)

$$
\left\{\begin{array} { l } 
{ \operatorname { m i n } _ { x } f ( x ) } \\
{ g ( x ) = 0 }
\end{array} \Leftrightarrow \left\{\begin{array}{l}
\min _{u, v} f(u, v) \\
u=\phi(v)
\end{array} \Leftrightarrow \min _{v} f(\phi(v), v),\right.\right.
$$

i.e. problem $P_{1}$ is (locally) equivalent to a unconstrained minimization problem. Therefore

$$
0=\nabla f\left(\phi\left(v^{\star}\right), v^{\star}\right)=\left(\frac{\partial f}{\partial u} \frac{\partial \phi}{\partial v}+\frac{\partial f}{\partial v}\right)_{x^{\star}}=\left(-\frac{\partial f}{\partial u}\left(\frac{\partial g}{\partial u}\right)^{-1} \frac{\partial g}{\partial v}+\frac{\partial f}{\partial v}\right)_{x^{\star}} .
$$

Setting

$$
\lambda^{\star}=\left(-\frac{\partial f}{\partial u}\left(\frac{\partial g}{\partial u}\right)^{-1}\right)_{x^{\star}}^{\prime}
$$

yields

$$
\begin{equation*}
\left(\frac{\partial f}{\partial v}+\left(\lambda^{\star}\right)^{\prime} \frac{\partial g}{\partial v}\right)_{x^{\star}}=0 \tag{3.9}
\end{equation*}
$$

[^11]and
\[

$$
\begin{equation*}
\left(\frac{\partial f}{\partial u}+\left(\lambda^{\star}\right)^{\prime} \frac{\partial g}{\partial u}\right)_{x^{\star}}=0 \tag{3.10}
\end{equation*}
$$

\]

Finally, let

$$
L=f+\lambda^{\prime} g,
$$

note that equations (3.9) and (3.10) can be rewritten as

$$
\nabla_{x} L\left(x^{\star}, \lambda^{\star}\right)=0
$$

and this, together with $g\left(x^{\star}\right)=0$, is equivalent to equations (3.7).

### 3.2.2 Quadratic cost function with linear equality constraints

Consider the function

$$
f(x)=\frac{1}{2} x^{\prime} Q x
$$

with $x \in \mathbb{R}^{n}$ and $Q=Q^{\prime}>0$, the equality constraints

$$
g(x)=A x-b=0,
$$

with $b \in \mathbb{R}^{m}$ and $m<n$, and the Lagrangian function

$$
L(x, \lambda)=\frac{1}{2} x^{\prime} Q x+\lambda^{\prime}(A x-b) .
$$

A simple application of Theorem 14 yields the necessary conditions of optimality

$$
\begin{align*}
\nabla_{x} L\left(x^{\star}, \lambda^{\star}\right) & =Q x^{\star}+A^{\prime} \lambda^{\star}=0 \\
g\left(x^{\star}\right) & =A x^{\star}-b=0 . \tag{3.11}
\end{align*}
$$

Suppose now that the matrix $A$ is such that $A Q^{-1} A^{\prime}$ is invertible ${ }^{4}$. As a result, the only solution of equations (3.11) is

$$
x^{\star}=Q^{-1} A^{\prime}\left(A Q^{-1} A^{\prime}\right)^{-1} b \quad \lambda^{\star}=-\left(A Q^{-1} A^{\prime}\right)^{-1} b .
$$

Finally, by Theorem 15, it follows that $x^{\star}$ is a strict constrained (global) minimum.

### 3.3 Nonlinear programming methods: introduction

The methods of non-linear programming that have been mostly studied in recent years belong to two categories. The former includes all methods based on the transformation of a constrained problem into one or more unconstrained problems, in particular the socalled (exact or sequential) penalty function methods and (exact or sequential) augmented Lagrangian methods. Sequential methods are based on the solution of a sequence of problems, with the property that the sequence of the solutions of the subproblems converge

[^12]to the solution of the original problem. Exact methods are based on the fact that, under suitable assumptions, the optimal solutions of an unconstrained problem coincides with the optimal solution of the original problem.
The latter includes the methods based on the transformation of the original problem into a sequence of constrained quadratic problems.
From the above discussion it is obvious that, to construct algorithms for the solution of non-linear programming problems, it is necessary to use efficient unconstrained optimization routines.
Finally, in any practical implementation, it is also important to quantify the complexity of the algorithms in terms of number and type of operations (inversion of matrices, differentiation, ...), and the speed of convergence. These issues are still largely open, and will not be addressed in these notes.

### 3.4 Sequential and exact methods

### 3.4.1 Sequential penalty functions

In this section we study the so-called external sequential penalty functions. This name is based on the fact that the solutions of the resulting unconstrained problems are in general not admissible. There are also internal penalty functions (known as barrier functions) but this can be used only for problems in which the admissible set has a non-empty interior. As a result, such functions cannot be used in the presence of equality constraints.
The basic idea of external sequential penalty functions is very simple. Consider problem $P_{0}$, the function

$$
q(x)= \begin{cases}0, & \text { if } x \in \mathcal{X}  \tag{3.12}\\ +\infty, & \text { if } x \notin \mathcal{X}\end{cases}
$$

and the function

$$
\begin{equation*}
F=f+q \tag{3.13}
\end{equation*}
$$

It is obvious that the unconstrained minimization of $F$ yields a solution of problem $P_{0}$. However, because of its discontinuous nature, the minimization of $F$ cannot be performed. Nevertheless, it is possible to construct a sequence of continuously differentiable functions, converging to $F$, and it is possible to study the convergence of the minima of such a sequence of functions to the solutions of problem $P_{0}$.
For, consider a continuously differentiable function $p$ such that

$$
p(x)= \begin{cases}0, & \text { if } x \in \mathcal{X}  \tag{3.14}\\ >0, & \text { if } x \notin \mathcal{X}\end{cases}
$$

and the function

$$
F_{\epsilon}=f+\frac{1}{\epsilon} p
$$

with $\epsilon>0$. It is obvious that ${ }^{5}$

$$
\lim _{\epsilon \rightarrow 0} F_{\epsilon}=F
$$

The function $F_{\epsilon}$ is known as external penalty function. The attribute external is due to the fact that, if $\bar{x}$ is a minimum of $F_{\epsilon}$ in general $p(\bar{x}) \neq 0$, i.e. $\bar{x} \notin \mathcal{X}$. The term $\frac{1}{\epsilon} p$ is called penalty term, as it penalizes the violation of the constraints. In general, the function $p$ has the following form

$$
\begin{equation*}
p=\sum_{i=1}^{m}\left(g_{i}\right)^{2}+\sum_{i=1}^{p}\left(\max \left(0, h_{i}\right)\right)^{2} \tag{3.15}
\end{equation*}
$$

Consider now a strictly decreasing sequence $\left\{\epsilon_{k}\right\}$ such that $\lim _{k \rightarrow \infty} \epsilon_{k}=0$. The sequential penalty function method consists in solving the sequence of unconstrained problems

$$
\min _{x} F_{\epsilon_{k}}(x),
$$

with $x \in \mathbb{R}^{n}$. The most important convergence results for this methods are summarized in the following statements.

Theorem 16 Consider the problem $P_{0}$. Suppose that for all $\sigma>0$ the set ${ }^{6}$

$$
\mathcal{X}^{\sigma}=\left\{x \in \mathbb{R}^{n}| | g_{i}(x) \mid \leq \sigma, i=1, \cdots, m\right\} \cap\left\{x \in \mathbb{R}^{n} \mid h_{i}(x) \leq \sigma, i=1, \cdots, p\right\}
$$

is compact. Suppose moreover that for all $k$ the function $F_{\epsilon_{k}}(x)$ has a global minimum $x_{k}$. Then the sequence $\left\{x_{k}\right\}$ has (at least) one converging subsequence, and the limit of any converging subsequence is a global minimum for problem $P_{0}$.

Theorem 17 Let $x^{\star}$ be a strict constrained local minimum for problem $P_{0}$. Then there exist a sequence $\left\{x_{k}\right\}$ and an integer $\bar{k}>0$ such that $\left\{x_{k}\right\}$ converges to $x^{\star}$ and, for all $k \geq \bar{k}, x_{k}$ is a local minimum of $F_{\epsilon_{k}}(x)$.

The construction of the function $F_{\epsilon}$ is apparently very simple, and this is the main advantage of the method. However, the minimization of the function $F_{\epsilon}$ may be difficult, especially for small values of $\epsilon$. In fact, it is possible to show, even via simple examples, that as $\epsilon$ tends to zero the Hessian matrix of the function $F_{\epsilon}$ becomes ill conditioned. As a result, any unconstrained minimization algorithm used to minimize $F_{\epsilon}$ has a very slow convergence rate. To alleviate this problem, it is possible to use, in the minimization of $F_{\epsilon_{k+1}}$, as initial point the point $x_{k}$. However, this is close to the minimum of $F_{\epsilon_{k+1}}$ only if $\epsilon_{k+1}$ is close to $\epsilon_{k}$, i.e. only if the sequence $\left\{\epsilon_{k}\right\}$ converges slowly to zero.
We conclude that, to avoid the ill conditioning of the Hessian matrix of $F_{\epsilon}$, hence the slow convergence of each unconstrained optimization problem, it is necessary to slow down the convergence of the sequence $\left\{x_{k}\right\}$, i.e. slow convergence is an intrinsic property of the method. This fact has motivated the search for alternatives methods, as described in the next sections.

[^13]Remark. It is possible to show that the local minima of $F_{\epsilon}$ describe (continuous) trajectories that can be extrapolated. This observation is exploited in some sophisticated methods for the selection of initial estimate for the point $x_{k}$. However, even with the addition of this extrapolation procedure, the convergence of the method remains slow.

Remark. Note that, if the function $p$ is defined as in equation (3.15), then the function $F_{\epsilon}$ is not two times differentiable everywhere, i.e. it is not differentiable in all points in which an inequality constraints is active. This property restricts the class of minimization algorithms that can be used to minimize $F_{\epsilon}$.

### 3.4.2 Sequential augmented Lagrangian functions

Consider problem $P_{1}$, i.e. an optimization problem with only equality constraints. For such a problem the Lagrangian function is

$$
L=f+\lambda^{\prime} g
$$

and the first order necessary conditions require the existence of a multiplier $\lambda^{\star}$ such that, for any local solution $x^{\star}$ of problem $P_{1}$ one has

$$
\begin{align*}
\nabla_{x} L\left(x^{\star}, \lambda^{\star}\right) & =0  \tag{3.16}\\
\nabla_{\lambda} L\left(x^{\star}, \lambda^{\star}\right) & =g\left(x^{\star}\right)=0
\end{align*}
$$

The first of equations (3.16) is suggestive of the fact that the function $L\left(x, \lambda^{\star}\right)$ has a unconstrained minimum in $x^{\star}$. This is actually not the case, as $L\left(x, \lambda^{\star}\right)$ is not convex in a neighborhood of $x^{\star}$. However it is possible to render the function $L\left(x, \lambda^{\star}\right)$ convex with the addition of a penalty term, yielding the new function, known as augmented Lagrangian function ${ }^{7}$,

$$
\begin{equation*}
L_{a}\left(x, \lambda^{\star}\right)=L\left(x, \lambda^{\star}\right)+\frac{1}{\epsilon}\|g(x)\|^{2} \tag{3.17}
\end{equation*}
$$

which, for $\epsilon$ sufficiently small, but such that $1 / \epsilon$ is finite, has a unconstrained minimum in $x^{\star}$. This intuitive discussion can be given a formal justification, as shown in the next statement.

Theorem 18 Suppose that at $x^{\star}$ and $\lambda^{\star}$ the sufficient conditions for a strict constrained local minimum for problem $P_{1}$ hold. Then there exists $\bar{\epsilon}>0$ such that for any $\epsilon \in(0, \bar{\epsilon})$ the point $x^{\star}$ is a unconstrained local minimum for the function $L_{a}\left(x, \lambda^{\star}\right)$.
Vice-versa, if for some $\bar{\epsilon}$ and $\lambda^{\star}$, at $x^{\star}$ the sufficient conditions for a unconstrained local minimum for the function $L_{a}\left(x, \lambda^{\star}\right)$ hold, and $g\left(x^{\star}\right)=0$, then $x^{\star}$ is a strict constrained local minimum for problem $P_{1}$.

The above theorem highlights the fact that, under the stated assumptions, the function $L_{a}\left(x, \lambda^{\star}\right)$ is an (external) penalty function, with the property that, to give local minima

[^14]for problem $P_{1}$ it is not necessary that $\epsilon \rightarrow 0$. Unfortunately, this result is not of practical interest, because it requires the knowledge of $\lambda^{\star}$. To obtain a useful algorithm, it is possible to make use of the following considerations.
By the implicit function theorem, applied to the first of equation (3.16), we infer that there exist a neighborhood of $\lambda^{\star}$, a neighborhood of $x^{\star}$, and a continuously differentiable function $x(\lambda)$ such that (locally)
$$
\nabla_{x} L_{a}(x(\lambda), \lambda)=0
$$

Moreover, for any $\epsilon \in(0, \bar{\epsilon})$, as $\nabla_{x x}^{2} L_{a}\left(x^{\star}, \lambda^{\star}\right)$ is positive definite also $\nabla_{x x}^{2} L_{a}(x, \lambda)$ is locally positive definite. As a result, $x(\lambda)$ is the only value of $x$ that, for any fixed $\lambda$, minimizes the function $L_{a}(x, \lambda)$. It is therefore reasonable to assume that if $\lambda_{k}$ is a good estimate of $\lambda^{\star}$, then the minimization of $L_{a}\left(x, \lambda_{k}\right)$ for a sufficiently small value of $\epsilon$, yields a point $x_{k}$ which is a good approximation of $x^{\star}$.
On the basis of the above discussion it is possible to construct the following minimization algorithm for problem $P_{1}$.

Step 0. Given $x_{0} \in \mathbb{R}^{n}, \lambda_{1} \in \mathbb{R}^{m}$ and $\epsilon_{1}>0$.
Step 1. Set $k=1$.
Step 2. Find a local minimum $x_{k}$ of $L_{a}\left(x, \lambda_{k}\right)$ using any unconstrained minimization algorithm, with starting point $x_{k-1}$.

Step 3. Compute a new estimate $\lambda_{k+1}$ of $\lambda^{\star}$.
Step 4. Set $\epsilon_{k+1}=\beta \epsilon_{k}$, with $\beta=1$ if $\left\|g\left(x_{k+1}\right)\right\| \leq \frac{1}{4}\left\|g\left(x_{k}\right)\right\|$ or $\beta<1$ otherwise.
Step 5. Set $k=k+1$ and go to Step 2.
In Step 3 it is necessary to construct a new estimate $\lambda_{k+1}$ of $\lambda_{k}$. This can be done with proper considerations on the function $L_{a}(x(\lambda), \lambda)$, introduced in the above discussion. However, without providing the formal details, we mention that one of the most used update laws for $\lambda$ are described by the equations

$$
\begin{equation*}
\lambda_{k+1}=\lambda_{k}+\frac{2}{\epsilon_{k}} g\left(x_{k}\right) \tag{3.18}
\end{equation*}
$$

or

$$
\begin{equation*}
\lambda_{k+1}=\lambda_{k}-\left[\nabla^{2} L_{a}\left(x\left(\lambda_{k}\right), \lambda_{k}\right)\right]^{-1} g\left(x_{k}\right), \tag{3.19}
\end{equation*}
$$

whenever the indicated inverse exists.
Note that the convergence of the sequence $\left\{x_{k}\right\}$ to $x^{\star}$ is limited by the convergence of the sequence $\left\{\lambda_{k}\right\}$ to $\lambda^{\star}$. It is possible to prove that, if the update law (3.18) is used then the algorithm as linear convergence, whereas if (3.19) is used the convergence is superlinear.
Remark. Similar considerations can be done for problem $P_{2}$. For, recall that problem $P_{2}$ can be recast, increasing the number of variables, as an optimization problem with equality
constraints, i.e. problem $\tilde{P}_{2}$. For such an extended problem, consider the augmented Lagrangian

$$
L_{a}(x, y, \rho)=f(x)+\sum_{i=1}^{p} \rho_{i}\left(h_{i}(x)+y_{i}^{2}\right)+\frac{1}{\epsilon} \sum_{i=1}^{p}\left(h_{i}(x)+y_{i}^{2}\right)^{2},
$$

and note that, in principle, it would be possible to make use of the results developed with reference to problem $P_{1}$. However, the function $L_{a}$ can be analytically minimized with respect to the variables $y_{i}$. In fact, a simple computation shows that the (global) minimum of $L_{a}$ as a function of $y$ is attained at

$$
y_{i}(x, \rho)=\sqrt{-\min \left(0, h_{i}(x)+\frac{\epsilon}{2} \rho_{i}\right)} .
$$

As a result, the augmented Lagrangian function for problem $P_{2}$ is given by

$$
L_{a}(x, \rho)=f(x)+\rho^{\prime} h(x)+\frac{1}{\epsilon}\|h(x)\|^{2}-\frac{1}{\epsilon} \sum_{i=1}^{p}\left(\min \left(0, h_{i}(x)+\frac{\epsilon}{2} \rho_{i}\right)\right)^{2} .
$$

### 3.4.3 Exact penalty functions

An exact penalty function, for a given constrained optimization problem, is a function of the same variables of the problem with the property that its unconstrained minimization yields a solution of the original problem. The term exact as opposed to sequential indicates that only one, instead of several, minimization is required.
Consider problem $P_{1}$, let $x^{\star}$ be a local solution and let $\lambda^{\star}$ be the corresponding multiplier. The basic idea of exact penalty functions methods is to determine the multiplier $\lambda$ appearing in the augmented Lagrangian function as a function of $x$, i.e. $\lambda=\lambda(x)$, with $\lambda\left(x^{\star}\right)=\lambda^{\star}$. With the use of this function one has ${ }^{8}$

$$
L_{a}(x, \lambda(x))=f(x)+\lambda(x)^{\prime} g(x)+\frac{1}{\epsilon}\|g(x)\|^{2} .
$$

The function $\lambda(x)$ is obtained considering the necessary condition of optimality

$$
\begin{equation*}
\nabla_{x} L_{a}\left(x^{\star}, \lambda^{\star}\right)=\nabla f\left(x^{\star}\right)+\frac{\partial g\left(x^{\star}\right)^{\prime}}{\partial x} \lambda^{\star}=0 \tag{3.20}
\end{equation*}
$$

and noting that, if $x^{\star}$ is a regular point for the constraints then equation (3.20) can be solved for $\lambda^{\star}$ yielding

$$
\lambda^{\star}=-\left(\frac{\partial g\left(x^{\star}\right)}{\partial x} \frac{\partial g\left(x^{\star}\right)^{\prime}}{\partial x}\right)^{-1} \frac{\partial g\left(x^{\star}\right)}{\partial x} \nabla f\left(x^{\star}\right)
$$

[^15]This equality suggests to define the function $\lambda(x)$ as

$$
\lambda(x)=-\left(\frac{\partial g(x)}{\partial x} \frac{\partial g(x)^{\prime}}{\partial x}\right)^{-1} \frac{\partial g(x)}{\partial x} \nabla f(x)
$$

and this is defined at all $x$ where the indicated inverse exists, in particular at $x^{\star}$.
It is possible to show that this selection of $\lambda(x)$ yields and exact penalty function for problem $P_{1}$. For, consider the function

$$
G(x)=f(x)-g(x)^{\prime}\left(\frac{\partial g(x)}{\partial x} \frac{\partial g(x)^{\prime}}{\partial x}\right)^{-1} \frac{\partial g(x)}{\partial x} \nabla f(x)+\frac{1}{\epsilon}\|g(x)\|^{2}
$$

which is defined and differentiable in the set

$$
\begin{equation*}
\tilde{\mathcal{X}}=\left\{x \in \mathbb{R}^{n} \| \operatorname{rank} \frac{\partial g(x)}{\partial x}=m\right\} \tag{3.21}
\end{equation*}
$$

and the following statements.
Theorem 19 Let $\overline{\mathcal{X}}$ be a compact subset of $\tilde{\mathcal{X}}$. Assume that $x^{\star}$ is the only global minimum of $f$ in $\mathcal{X} \cap \overline{\mathcal{X}}$ and that $x^{\star}$ is in the interior of $\overline{\mathcal{X}}$. Then there exists $\bar{\epsilon}>0$ such that, for any $\epsilon \in(0, \bar{\epsilon}), x^{\star}$ is the only global minimum of $G$ in $\overline{\mathcal{X}}$.

Theorem 20 Let $\overline{\mathcal{X}}$ be a compact subset of $\tilde{\mathcal{X}}$. Then there exists $\bar{\epsilon}>0$ such that, for any $\epsilon \in(0, \bar{\epsilon})$, if $x^{\star}$ is a unconstrained minimum of $G(x)$ and $x^{\star} \in \overline{\mathcal{X}}$, then $x^{\star}$ is a constrained local minimum for problem $P_{1}$.

Theorems 19 and 20 show that it is legitimate to search solutions of problem $P_{1}$ minimizing the function $G$ for sufficiently small values of $\epsilon$. Note that it is possible to prove stronger results, showing that there is (under certain hypotheses) a one to one correspondence between the minima of problem $P_{1}$ and the minima of the function $G$, provided $\epsilon$ is sufficiently small.
For problem $P_{2}$ it is possible to proceed as discussed in Section 3.4.2, i.e. transforming problem $P_{2}$ into problem $\tilde{P}_{2}$ and then minimizing analytically with respect to the new variables $y_{i}$. However, a different and more direct route can be taken. Consider problem $P_{2}$ and the necessary conditions

$$
\begin{equation*}
\nabla_{x} L\left(x^{\star}, \rho^{\star}\right)=\nabla f\left(x^{\star}\right)+\frac{\partial h\left(x^{\star}\right)^{\prime}}{\partial x} \rho^{\star}=0 \tag{3.22}
\end{equation*}
$$

and

$$
\begin{equation*}
\rho_{i}^{\star} h_{i}\left(x^{\star}\right)=0, \tag{3.23}
\end{equation*}
$$

for $i=1, \cdots, p$. Premultiplying equation (3.22) by $\frac{\partial h\left(x^{\star}\right)}{\partial x}$ and equation (3.23) by $\gamma^{2} h_{i}\left(x^{\star}\right)$, with $\gamma>0$, and adding, yields

$$
\left(\frac{\partial h\left(x^{\star}\right)}{\partial x} \frac{\partial h\left(x^{\star}\right)^{\prime}}{\partial x}+\gamma^{2} H^{2}\left(x^{\star}\right)\right) \rho^{\star}+\frac{\partial h\left(x^{\star}\right)}{\partial x} \nabla f\left(x^{\star}\right)=0,
$$

where

$$
H\left(x^{\star}\right)=\operatorname{diag}\left(h_{1}\left(x^{\star}\right), \cdots, h_{p}\left(x^{\star}\right)\right)
$$

As a result, a natural candidate for the function $\rho(x)$ is

$$
\rho(x)=-\left(\frac{\partial h(x)}{\partial x} \frac{\partial h(x)^{\prime}}{\partial x}+\gamma^{2} H^{2}(x)\right)^{-1} \frac{\partial h(x)}{\partial x} \nabla f(x)
$$

which is defined whenever the indicated inverse exists, in particular in the neighborhood of any regular point. With the use of this function, it is possible to define an exact penalty function for problem $P_{2}$ and to establish results similar to those illustrated in Theorems 19 and 20.
The exact penalty functions considered in this section provide, in principle, a theoretically sound way of solving constrained optimization problem. However, in practice, they have two major drawbacks. Firstly, at each step, it is necessary to invert a matrix with dimension equal to the number of constraint. This operation is numerically ill conditioned if the number of constraints is large. Secondly, the exact penalty functions may not be sufficiently regular to allow the use of unconstrained minimization methods with fast speed of convergence, e.g. Newton method.

### 3.4.4 Exact augmented Lagrangian functions

An exact augmented Lagrangian function, for a given constrained optimization problem, is a function, defined on an augmented space with dimension equal to the number of variables plus the number of constraint, with the property that its unconstrained minimization yields a solution of the original problem. Moreover, in the computation of such a function it is not necessary to invert any matrix.
To begin with, consider problem $P_{1}$ and recall that, for such a problem, a sequential augmented Lagrangian function has been defined adding to the Lagrangian function a term, namely $\frac{1}{\epsilon}\|g(x)\|^{2}$, which penalizes the violation of the necessary condition $g(x)=0$. This term, for $\epsilon$ sufficiently small, renders the function $L_{a}$ convex in a neighborhood of $x^{\star}$. A complete convexification can be achieved adding a further term that penalizes the violation of the necessary condition $\nabla_{x} L(x, \lambda)=0$. More precisely, consider the function

$$
\begin{equation*}
S(x, \lambda)=f(x)+\lambda^{\prime} g(x)+\frac{1}{\epsilon}\|g(x)\|^{2}+\eta\left\|\frac{\partial g(x)}{\partial x} \nabla_{x} L(x, \lambda)\right\|^{2} \tag{3.24}
\end{equation*}
$$

with $\epsilon>0$ and $\eta>0$. The function (3.24) is continuously differentiable and it is such that, for $\epsilon$ sufficiently small, the solutions of problem $P_{1}$ are in a one to one correspondence with the points $(x, \lambda)$ which are local minima of $S$, as detailed in the following statements.

Theorem 21 Let $\overline{\mathcal{X}}$ be a compact set. Suppose $x^{\star}$ is the unique global minimum of $f$ in the set $\mathcal{X} \cap \overline{\mathcal{X}}$ and $x^{\star}$ is an interior point of $\overline{\mathcal{X}}$. Let $\lambda^{\star}$ be the multiplier associated to $x^{\star}$. Then, for any compact set $\Lambda \subset \mathbb{R}^{m}$ such that $\lambda^{\star} \in \Lambda$ there exists $\bar{\epsilon}$ such that, for all $\epsilon \in(0, \bar{\epsilon}),\left(x^{\star}, \lambda^{\star}\right)$ is the unique global minimum of $S$ in $\mathcal{X} \times \Gamma$.

Theorem 22 Let $^{9} \mathcal{X} \times \Lambda \subset \tilde{\mathcal{X}} \times \mathbb{R}^{m}$ be a compact set. Then there exists $\bar{\epsilon}>0$ such that, for all $\epsilon \in(0, \bar{\epsilon})$, if $\left(x^{\star}, \lambda^{\star}\right)$ is a unconstrained local minimum of $S$, then $x^{\star}$ is a constrained local minimum for problem $P_{1}$ and $\lambda^{\star}$ is the corresponding multiplier.

Theorems 21 and 22 justify the use of a unconstrained minimization algorithm, applied to the function $S$, to find local (or global) solutions of problem $P_{1}$.
Problem $P_{2}$ can be dealt with using the same considerations done in Section 3.4.2.

### 3.5 Recursive quadratic programming

Recursive quadratic programming methods have been widely studied in the past years. In this section we provide a preliminary description of such methods. For, consider problem $P_{1}$ and suppose that $x^{\star}$ and $\lambda^{\star}$ are such that the necessary conditions (3.7) hold. Consider now a series expansion of the function $L\left(x, \lambda^{\star}\right)$ in a neighborhood of $x^{\star}$, i.e.

$$
L\left(x, \lambda^{\star}\right)=f\left(x^{\star}\right)+\frac{1}{2}\left(x-x^{\star}\right)^{\prime} \nabla_{x x}^{2} L\left(x^{\star}, \lambda^{\star}\right)\left(x-x^{\star}\right)+\ldots
$$

a series expansion of the constraint, i.e.

$$
0=g(x)=g\left(x^{\star}\right)+\frac{\partial g\left(x^{\star}\right)}{\partial x}\left(x-x^{\star}\right)+\ldots
$$

and the problem

$$
\widetilde{P Q}_{1}\left\{\begin{array}{l}
\min _{x} f\left(x^{\star}\right)+\frac{1}{2}\left(x-x^{\star}\right)^{\prime} \nabla_{x x}^{2} L\left(x^{\star}, \lambda^{\star}\right)\left(x-x^{\star}\right) \\
\frac{\partial g\left(x^{\star}\right)}{\partial x}\left(x-x^{\star}\right)=0
\end{array}\right.
$$

Note that problem $\widetilde{P Q}_{1}$ has the solution $x^{\star}$, and the corresponding multiplier is $\lambda=0$, which is not equal (in general) to $\lambda^{\star}$. This phenomenon is called bias of the multiplier, and can be avoided by modifying the objective function and considering the new problem

$$
P Q_{1}\left\{\begin{array}{l}
\min _{x} f\left(x^{\star}\right)+\nabla f\left(x^{\star}\right)^{\prime}\left(x-x^{\star}\right)+\frac{1}{2}\left(x-x^{\star}\right)^{\prime} \nabla_{x x}^{2} L\left(x^{\star}, \lambda^{\star}\right)\left(x-x^{\star}\right)  \tag{3.25}\\
\frac{\partial g\left(x^{\star}\right)}{\partial x}\left(x-x^{\star}\right)=0
\end{array}\right.
$$

which has solution $x^{\star}$ with multiplier $\lambda^{\star}$. This observation suggests to consider the sequence of quadratic programming problems

$$
P Q_{1}^{k+1}\left\{\begin{array}{l}
\min _{\delta} f\left(x_{k}\right)+\nabla f\left(x_{k}\right)^{\prime} \delta+\frac{1}{2} \delta^{\prime} \nabla_{x x}^{2} L\left(x_{k}, \lambda_{k}\right) \delta  \tag{3.26}\\
\frac{\partial g\left(x_{k}\right)}{\partial x} \delta=0
\end{array}\right.
$$

[^16]where $\delta=x-x_{k}$, and $x_{k}$ and $\lambda_{k}$ are the current estimates of the solution and of the multiplier. The solution of problem $P Q_{1}^{k+1}$ yields new estimates $x_{k+1}$ and $\lambda_{k+1}$. To assess the local convergence of the method, note that the necessary conditions for problem $P Q_{1}^{k+1}$ yields the system of equations
\[

\left[$$
\begin{array}{cc}
\nabla_{x x}^{2} L\left(x_{k}, \lambda_{k}\right) & \frac{\partial g\left(x_{k}\right)^{\prime}}{\partial x}  \tag{3.27}\\
\frac{\partial g\left(x_{k}\right)}{\partial x} & 0
\end{array}
$$\right]\left[$$
\begin{array}{c}
\delta \\
\lambda
\end{array}
$$\right]=-\left[$$
\begin{array}{c}
\nabla f\left(x_{k}\right) \\
g\left(x_{k}\right)
\end{array}
$$\right]
\]

and this system coincides with the system arising from the application of Newton method to the solution of the necessary conditions for problem $P_{1}$. As a consequence, the solutions of the problems $P Q_{1}^{k+1}$ converge to a solution of problem $P_{1}$ under the same hypotheses that guarantee the convergence of Newton method.

Theorem 23 Let $x^{\star}$ be a strict constrained local minimum for problem $P_{1}$, and let $\lambda^{\star}$ be the corresponding multiplier. Suppose that for $x^{\star}$ and $\lambda^{\star}$ the sufficient conditions of Theorem 15 hold. Then there exists an open neighborhood $\Omega \subset \mathbb{R}^{n} \times \mathbb{R}^{m}$ of the point $\left(x^{\star}, \lambda^{\star}\right)$ such that, if $\left(x_{0}, \lambda_{0}\right) \in \Omega$, the sequence $\left\{x_{k}, \lambda_{k}\right\}$ obtained solving the sequence of quadratic programming problems $P Q_{1}^{k+1}$, with $k=0,1, \cdots$, converges to $\left(x^{\star}, \lambda^{\star}\right)$.
Moreover, the speed of convergence is superlinear, and, if $f$ and $g$ are three times differentiable, the speed of convergence is quadratic.

Remark. It is convenient to solve the sequence of quadratic programming problems $P Q_{1}^{k+1}$, instead of solving the equations (3.27) with Newton method, because, for the former it is possible to exclude converge to maxima or saddle points.

In the case of problem $P_{2}$, using considerations similar to the one above, it is easy to obtain the following sequence of quadratic programming problems

$$
P Q_{2}^{k+1}\left\{\begin{array}{l}
\min _{\delta} f\left(x_{k}\right)+\nabla f\left(x_{k}\right)^{\prime} \delta+\frac{1}{2} \delta^{\prime} \nabla_{x x}^{2} L\left(x_{k}, \lambda_{k}\right) \delta  \tag{3.28}\\
\frac{\partial h\left(x_{k}\right)}{\partial x} \delta+h\left(x_{k}\right) \leq 0
\end{array}\right.
$$

This sequence of problems has to be solved iteratively to generate a sequence $\left\{x_{k}, \lambda_{k}\right\}$ that, under hypotheses similar to those of Theorem 23, converges to a strict constrained local minimum of problem $P_{2}$.
The method described are the basis for a large class of iterative methods.
A first disadvantage of the proposed iterative schemes is that it is necessary to compute the second derivatives of the functions of the problem. This computation can be avoided, using the same philosophy of quasi-Newton methods.
A second disadvantage is in the fact that, being based on Newton algorithm, only local convergence can be guaranteed. However, it is possible to combine the method with global convergent methods: these are used to generate a pair $(\tilde{x}, \tilde{\lambda})$ sufficiently close to $\left(x^{\star}, \lambda^{\star}\right)$
and then recursive quadratic programming methods are used to obtain fast convergence to $\left(x^{\star}, \lambda^{\star}\right)$.
A third disadvantage is in the fact that there is no guarantee that the sequence of admissible sets generated by the algorithm is non-empty at each step.
Finally, it is worth noting that, contrary to most of the existing methods, quadratic programming methods do not rely on the use of a penalty term.
Remark. There are several alternative recursive quadratic programming methods which alleviate the drawbacks of the methods described. These are (in general) based on the use of quadratic approximation of penalty functions. For brevity, we do not discuss these methods.
$\diamond$

### 3.6 Concluding remarks

In this section we briefly summarize advantages and disadvantages of the nonlinear programming methods discussed.
Sequential penalty functions methods are very simple to implement, but suffer from the ill conditioning associated to large penalties (i.e. to small values of $\epsilon$ ). As a result, these methods can be used if approximate solutions are acceptable, or in the determination of initial estimates for more precise, but only locally convergent, methods. Note, in fact, that not only an approximation of the solution $x^{\star}$ can be obtained, but also an approximation of the corresponding multiplier $\lambda^{\star}$. For example, for problem $P_{1}$, a (approximate) solution $\bar{x}$ is such that

$$
\nabla F_{\epsilon_{k}}(\bar{x})=\nabla f(\bar{x})+\frac{2}{\epsilon_{k}} \frac{\partial g(\bar{x})}{\partial x} g(\bar{x})=0
$$

hence, the term $\frac{2}{\epsilon_{k}} g(\bar{x})$ provides an approximation of $\lambda^{\star}$.
Sequential augmented Lagrangian functions do not suffer from ill conditioning, and yield faster speed of convergence then that attainable using sequential penalty functions.
The methods based on exact penalty functions do not require the solution of a sequence of problems. However, they require the inversion of a matrix of dimension equal to the number of constraints, hence their applicability is limited to problems with a small number of constraints.
Exact augmented Lagrangian functions can be built without inverting any matrix. However, the minimization has to be performed in an extended space.
Recursive quadratic programming methods require the solution, at each step, of a constrained quadratic programming problem. Their main problem is that there is no guarantee that the admissible set is non-empty at each step.
We conclude that it is not possible to decide which is the best method. Each method has its own advantages and disadvantages. Therefore, the selection of a nonlinear programming method has to be driven by the nature of the problem: and has to take into consideration the number of variables, the regularity of the involved functions, the required precision, the computational cost, ....

## Chapter 4

## Global optimization

### 4.1 Introduction

Given a function $f: \mathbb{R}^{n} \rightarrow \mathbb{R}$, global optimization methods aim at finding the global minimum of $f$, i.e. a point $x^{\star}$ such that

$$
f\left(x^{\star}\right) \leq f(x)
$$

for all $x \in \mathbb{R}^{n}$. Among these methods it is possible to distinguish between deterministic methods and probabilistic methods.
In the following sections we provide a very brief introductions to global minimization methods. It is worth noting that this is an active area of research.

### 4.2 Deterministic methods

### 4.2.1 Methods for Lipschitz functions

Consider a function $f: \mathbb{R}^{n} \rightarrow \mathbb{R}$ and suppose it is Lipschitz with constant $L>0$, i.e.

$$
\begin{equation*}
\left|f\left(x_{1}\right)-f\left(x_{2}\right)\right| \leq L\left\|x_{1}-x_{2}\right\|, \tag{4.1}
\end{equation*}
$$

for all $x_{1} \in \mathbb{R}^{n}$ and $x_{2} \in \mathbb{R}^{n}$. Note that equation (4.1) implies that

$$
\begin{equation*}
f(x) \geq f\left(x_{0}\right)-L\left\|x-x_{0}\right\| \tag{4.2}
\end{equation*}
$$

and that

$$
\begin{equation*}
f(x) \leq f\left(x_{0}\right)+L\left\|x-x_{0}\right\|, \tag{4.3}
\end{equation*}
$$

for all $x \in \mathbb{R}^{n}$ and $x_{0} \in \mathbb{R}^{n}$, see Figure 4.1 for a geometrical interpretation.


Figure 4.1: Geometrical interpretation of the Lipschitz conditions (4.2) and (4.3).

Methods for Lipschitz functions are suitable to find a global solution of the problem

$$
\min _{x} f(x)
$$

with

$$
x \in I_{n}=\left\{x \in \mathbb{R}^{n} \mid A_{i} \leq x_{i} \leq B_{i}\right\}
$$

and $A_{i}<B_{i}$ given, under the assumptions that the set $I_{n}$ contains a global minimum of $f$, the function $f$ is Lipschitz in $I_{n}$ and the Lipschitz constant $L$ of $f$ in $I_{n}$ is known. Under these assumptions it is possible to construct a very simple global minimization algorithm, known as Schubert-Mladineo algorithm, as follows.

Step 0. Given $x_{0} \in I_{n}$ and $\tilde{L}>L$.
Step 1. Set $k=0$.
Step 2. Let

$$
F_{k}(x)=\max _{j=0, \cdots, k}\left\{f\left(x_{j}\right)-\tilde{L}\left\|x-x_{j}\right\|\right\}
$$

and compute $x_{k+1}$ such that

$$
F_{k}\left(x_{k+1}\right)=\min _{x \in I_{n}} F_{k}(x) .
$$

Step 4. Set $k=k+1$ and go to Step 2.
Remark. The functions $F_{k}$ in Step 2 of the algorithm have a very special form. This can be exploited to construct special algorithms solving the problem

$$
\min _{x \in I_{n}} F_{k}(x)
$$

in a finite number of iterations.
For Schubert-Mladineo algorithm it is possible to prove the following statement.
Theorem 24 Let $f^{\star}$ be the minimum value of $f$ in $I_{n}$, let $x^{\star}$ be such that $f\left(x^{\star}\right)=f^{\star}$ and let $F_{k}^{\star}$ be the minima of the functions $F_{k}$ in $I_{n}$. Let

$$
\Phi=\left\{x \in I_{n} \mid f(x)=f^{\star}\right\}
$$

and let $\left\{x_{k}\right\}$ be the sequence generated by the algorithm. Then

- $\lim _{k \rightarrow \infty} f\left(x_{k}\right)=f^{\star}$;
- the sequence $\left\{F_{k}^{\star}\right\}$ is non-decreasing and $\lim _{k \rightarrow \infty} F_{k}^{\star}=f^{\star}$;
- $\lim _{k \rightarrow \infty} \inf _{x \in \Phi}\left\|x-x_{k}\right\|=0$;
- $f\left(x_{k}\right) \geq f^{\star} \geq F_{k-1}\left(x_{k}\right)$.

Schubert-Mladineo algorithm can be given, if $x \in I_{1} \subset \mathbb{R}$, a simple geometrical interpretation, as shown in Figure 4.2.
The main advantage of Schubert-Mladineo algorithm is that it does not require the computation of derivatives, hence it is also applicable to functions which are not everywhere


Figure 4.2: Geometrical interpretation of Schubert-Mladineo algorithm.
differentiable. Moreover, unlike other global minimization algorithms, it is possible to prove the convergence of the sequence $\left\{x_{k}\right\}$ to the global minimum. Finally, it is possible to define a simple stopping condition. For, note that if $\left\{x_{k}\right\}$ and $\left\{F_{k}^{\star}\right\}$ are the sequences generated by the algorithm, then

$$
f\left(x_{k}\right) \geq f^{\star} \geq F_{k}^{\star}
$$

and

$$
f\left(x_{k}\right) \geq f^{\star} \geq f\left(x_{k}\right)+r_{k},
$$

where $r_{k}=F_{k}^{\star}-f\left(x_{k}\right)$ and $\lim _{k \rightarrow \infty} r_{k}=0$. As a result, if $\left|r_{k}\right|<\epsilon$, for some $\epsilon>0$, the point $x_{k}$ gives a good approximation of the minimum of $f$.
The main disadvantage of the algorithm is in the assumption that the set $I_{n}$ contains a global minimum of $f$ in $\mathbb{R}^{n}$. Moreover, it may be difficult to compute the Lipschitz constant $L$.

### 4.2.2 Methods of the trajectories

The basic idea of the global optimization methods known as methods of the trajectories is to construct trajectories which go through all local minima. Once all local minima are determined, the global minimum can be easily isolated. These methods have been originally proposed in the 70 's, but only recently, because of increased computer power and of a reformulation using tools from differential geometry, they have proved to be effective.
The simplest and first method of the trajectories is the so-called Branin method. Consider the function $f$ and assume $\nabla f$ is continuous. Fix $x_{0}$ and consider the differential equations

$$
\begin{equation*}
\frac{d}{d t} \nabla f(x(t))= \pm \nabla f(x(t)) \quad x(0)=x_{0} \tag{4.4}
\end{equation*}
$$

The solutions $x(t)$ of such differential equations are such that

$$
\nabla f(x(t))=\nabla f\left(x_{0}\right) e^{ \pm t}
$$

i.e. $\nabla f(x(t))$ is parallel to $\nabla f\left(x_{0}\right)$ for all $t$. Using these facts it is possible to describe Branin algorithm.

Step 0. Given $x_{0}$.
Step 1. Compute the solution $x(t)$ of the differential equation

$$
\frac{d}{d t} \nabla f(x(t))=-\nabla f(x(t))
$$

with $x(0)=x_{0}$.
Step 2. The point $x^{\star}=\lim _{t \rightarrow \infty} x(t)$ is a stationary point of $f$, in fact $\lim _{t \rightarrow \infty} \nabla f(x(t))=0$.
Step 3. Consider a perturbation of the point $x^{\star}$, i.e. the point $\tilde{x}=x^{\star}+\epsilon$ and compute the solution $x(t)$ of the differential equation

$$
\frac{d}{d t} \nabla f(x(t))=\nabla f(x(t))
$$

Along this trajectory the gradient $\nabla f(x(t))$ increases, hence the trajectory escapes from the region of attraction of $x_{0}$.

Step 4. Fix $\bar{t}>0$ and assume that $x(\bar{t})$ is sufficiently away from $x_{0}$. Set $x_{0}=x(\bar{t})$ and go to Step 1.

Note that, if the perturbation $\epsilon$ and the time $\bar{t}$ are properly selected, at each iteration the algorithm generates a new stationary point of the function $f$.
Remark. If $\nabla^{2} f$ is continuous then the differential equations (4.4) can be written as

$$
\dot{x}(t)= \pm\left[\nabla^{2} f(x(t))\right]^{-1} \nabla f(x(t)) .
$$

Therefore Branin method is a continuous equivalent of Newton method. Note however that, as $\nabla^{2} f(x(t))$ may become singular, the above equation may be meaningless. In such a case it is possible to modify Branin method using ideas borrowed from quasi-Newton algorithms.

Branin method is very simple to implement. However, it has several disadvantages.

- It is not possible to prove convergence to the global minimum.
- Even if the method yields the global minimum, it is not possible to know how many iterations are needed to reach such a global minimum, i.e. there is no stopping criterion.
- The trajectories $x(t)$ are attracted by all stationary points of $f$, i.e. both minima and maxima.
- There is not a systematic way to select $\epsilon$ and $\bar{t}$.


Figure 4.3: Interpretation of the tunneling phase.

### 4.2.3 Tunneling methods

Tunneling methods have been proposed to find, in an efficient way, the global minimum of a function with several (possibly thousands) of local minima.
Tunneling algorithms are composed of a sequence of cycles, each having two phases. The first phase is the minimization phase, i.e. a local minimum is computed. The second phase is the tunneling phase, i.e. a new starting point for the minimization phase is computed.

## Minimization phase

Given a point $x_{0}$, a local minimization, using any unconstrained optimization algorithm, is performed. This minimization yields a local minimum $x_{0}^{\star}$.

## Tunneling phase

A point $x_{1} \neq x_{0}^{\star}$ such that

$$
f\left(x_{1}\right)=f\left(x_{0}^{\star}\right)
$$

is determined. See Figure 4.3 for a geometrical interpretation.

In theory, tunneling methods generate a sequence $\left\{x_{k}^{\star}\right\}$ such that

$$
f\left(x_{k+1}^{\star}\right) \leq f\left(x_{k}^{\star}\right)
$$



Figure 4.4: The functions $f(x)$ and $T\left(x, x_{k}^{\star}\right)$.
and the sequence $\left\{x_{k}^{\star}\right\}$ converges to the global minimum without passing through all local minima. This is the most important advantage of tunneling methods. The main disadvantage is the difficulty in performing the tunneling phase. In general, given a point $x_{k}^{\star}$ a point $x$ such that $f(x)=f\left(x_{k}^{\star}\right)$ is constructed searching for a zero of the function (see Figure 4.4)

$$
T\left(x, x_{k}^{\star}\right)=\frac{f(x)-f\left(x_{k}^{\star}\right)}{\left\|x-x_{k}^{\star}\right\|^{2 \lambda}},
$$

where the parameter $\lambda>0$ has to be selected such that $T\left(x_{k}^{\star}, x_{k}^{\star}\right)>0$.
Finally, it is worth noting that tunneling methods do not have a stopping criterion, i.e. the algorithm attempts to perform the tunneling phase even if the point $x_{k}^{\star}$ is a global minimum.

### 4.3 Probabilistic methods

### 4.3.1 Methods using random directions

In this class of algorithms at each iteration a randomly selected direction, having unity norm, is selected. The theoretical justification of such an algorithm rests on Gaviano theorem. This states that the sequence $\left\{x_{k}\right\}$ generated using the iteration

$$
x_{k+1}=x_{k}+\alpha_{k} d_{k},
$$

where $d_{k}$ is randomly selected on a unity norm sphere and $\alpha_{k}$ is such that

$$
f\left(x_{k}+\alpha_{k} d_{k}\right)=\min _{\alpha} f\left(x_{k}+\alpha d_{k}\right),
$$

is such that for any $\epsilon>0$ the probability that

$$
f\left(x_{k}\right)-f^{\star}<\epsilon,
$$

where $f^{\star}$ is a global minimum of $f$, tends to one as $k \rightarrow \infty$.

### 4.3.2 Multistart methods

Multistart methods are based on the fact that for given sets $D$ and $A$, with measures $m(D)$ and $m(A)$, and such that

$$
1 \geq \frac{m(A)}{m(D)}=\alpha \geq 0
$$

the probability that, selecting $N$ random points in $D$, one of these points is in $A$ is

$$
P(A, N)=1-(1-\alpha)^{N}
$$

As a result

$$
\lim _{N \rightarrow \infty} P(A, N)=1
$$

Therefore, if $A$ is a neighborhood of a global minimum of $f$ in $D$, we conclude that, selecting a sufficiently large number of random points in $D$, one of these will (almost surely) be close to the global minimum. Using these considerations it is possible to construct a whole class of algorithms, with similar properties, as detailed hereafter.

Step 0. Set $f^{\star}=\infty$.
Step 1. Select a random point $x_{0} \in \mathbb{R}^{n}$.
Step 2. If $f\left(x_{0}\right)>f^{\star}$ go to Step 1.
Step 3. Perform a local minimization starting from $x_{0}$ and yielding a point $x_{0}^{\star}$. Set $f^{\star}=f\left(x_{0}^{\star}\right)$.

Step 4. Check if $x_{0}^{\star}$ satisfies a stopping criterion. If not, go to Step 1.

### 4.3.3 Stopping criteria

The main disadvantage of probabilistic algorithms is the lack of a theoretically sound stopping criterion. The most promising and used stopping criterion is based on the construction of a probabilistic approximation $\tilde{P}(w)$ of the function

$$
P(w)=\frac{m(\{x \in D \mid f(x) \leq w\})}{m(D)} .
$$

Once the function $\tilde{P}(w)$ is known, a point $x^{\star}$ is regarded as a good approximation of the global minimum of $f$ if

$$
\tilde{P}\left(f\left(x^{\star}\right)\right) \leq \epsilon \ll 1 .
$$

## MAM SCHOOL OF ENGINEERING

Siruganur, Tiruchirappalli-621 105.

## Department of Mechatronics Engineering

## Teacher Teach Teacher (TTT)

Academic year (2019-2020) Even semester
Date: $\mathbf{1 2 . 3 . 2 0 2 0}$
Speaker: Mrs.P.Sudha
HOD - Mechatronics Engineering

Staff attended:

1. Dr.Punitha
2. Mr.M.Chandra sekar
3. Ms.M.Suba pradha
4. Mr.Arumugasamy
5. Mr.Revichandran
6. Mr.Karthikeyan

Topic:
Blooms Taxonomy

## Venue:

Smart class
Date \& Time:
$12^{\text {th }}$ March 2020 \& 2.00 P.M to 3.00 P.M
**enclosure: Report


## REPORT

The sestion was initiated by Mrs.P.Sudha HOD Mas, the moghe for that Lecture is Blor ass Taxonomy and discuss about the following topies

- Introduction
- Stage or Bloom's Taxonomy
- Bloom's verb chars
- Course level and lesson level objectives
- Steps towards writing effective leaming objectives
- Uses of Hloom's Taxonomy

The ses on comes to an end with the explaining the ovenvien Blama Taxonomy


1047

## Introduction

- Bloom's Taxonomy is a popular and extreprelylatipitil topat that is used by most teachers.


## Blooms Taxonomy

## Old and New version of Blooms <br> Taxonomy



New Version


## Contd..

- Like other taxonomies, Bloom's is hictarehicat, bieanitg that learning at the higher levels is dependent on/thaving atrained prerequisite knowledge and skills at lower letels, ff ith
- Pyramid into a "cake-style" hierarchy to smphitsize toigt anch level is built on a foundation of the previous terels.
1.Remembering: Retrieving, recognizing., and tecalling relevant knowledge from long-term memory.
2.Understanding: Constructing meaning from ocal, writitn, and graphic messages through interpreting, exemplifying, eltissifying. summarizing, inferring. comparing, and explainings
3.Applying: Carrying out or using a procedure for exespuing, or implementing.
4.Analyzing: Breaking material into constituent parts, detennining how the parts relate to one another and to an overall struerure or purpose through differentiating, organizing, and attributing.
5.Evaluating: Making judgments based on criteria and stinclards through cheeking and critiquing.
6.Creating: Putting elements together to form a coherent or functional whole; reorganizing elements into a new pattern or structure through generating, planning, or producing.


## Stages of Blooms Taxonomy

- Bloom's Taxonomy is a list of cognitive skills phat es cace by teachers to determine the level of thinking their ithedents have achieved.
- The taxonomy ranks the cognitive skills on a contimue from lower-order thinking to higher-order thinking.
- The taxonomy is often depicted by a pyramid that showis the hierarchy of cognitive skills.
- It was created by psychologist Benjamin Bloorn and scystal of his colleagues in 1948.
- It was then updated in the 1990 s by one of his shadentsinsmed Lorin Anderson.
- Anderson updated the names of the categories hind swapped the top two elements on the pyramid.



## How Bloom's can aid in course design

- Bloon's taxonomy is a powerful tool to help develop Iearining objectives because it explains the process of learning:
- Befarel you can understand a concept, you mustremember it.
${ }^{-} \mathrm{T}$ To apply alconcept you must first understand it.
In beder to evaluate a process, you must
heve analyed it.
To create ann faccurate conclusion, you must have
completed a thorough evaluation.

Contd..



## Coutse level and lesson level objectives

-The blggestlifference between course and lesson level objectives is that we do ${ }^{\circ} t$ directiy assess course level objectives.

- Course level objectives are just too broad.
-Instead! severallesson level objectives are used to demonstrate masteryiof one course level objective.
- To create good colirse level objectives, we need to ask ourselves: "what ad 1 want't the students to have mastery of at the end of the coursc H :
Then, fafter finalize our course level objectives, we have to make sure that mastery of all of the lesson level objectives underneath confirmi hat a student has mastery of the course level objective
-In othe twords, if students can prove (through assessment) that they can doleach and évery, one of the lesson level objectives in that section then asian instructor agree they have mastery of the course level objective.

Bloom's verb charts

| Hroundiag | 1. 5 | - |
| :---: | :---: | :---: |
| Create | design, forma s.as build. invent, cresti, on piphe, gencrate, derne, wodify. develop. |  1. Erictic.i. in ar ariginal homewerk. -- men: 'ry wat ise prencipla of <br>  |
| Evaluate | choose, surpyty, stas, determine, de fiss julge. grade, cenppex., intrat, argux, justify. wh mot, convince, schwt. vahate. |  ** thie to ct amine alocther using <br>  <br>  <br> 1. Ivन a a cy yaritics problem. |
| Analyze | classify, breal dy in. catcgotive, an ly ¿celagro. illustrate critice simpl it asseciate. | It Temit of his iswon, the stukiant with <br> Lit T, 1 lementic between potertial I koots wortay |

## How Bloom's works with course level and lesson level objectives:

- Course level objectives are broed. You may only have 3-5 course level objectives.
- They would be diflicult to meteure directly because they overarch the topics of your entire course.
- Lesson level objectives are what we use to demonstrate that a student has mastery of the course level objectives.
- We do this by building lesson level objectives that build toward the course level objective.


## Steps towards writing effective learning objectives:

1. Make sure there is onc mensurable verb in cach objective.
2. Each objective necds one verb. Either a student can master the objective, or they fail to master it. If an objective has two verbs (say, define and apply), what happens if a student can define, but not apply? Are they demonstrating mastery?
3. Ensure that the verbs in the course level objective are at least at the highest Blwom's Taxonemy as the highest Iesson level objectives that support it. (Because we can't verify they can evaluate if our lessons only tanght them (and assessed)
to define.)
4. Strive to keep all your learning objectives measurable, clear and concise.

## COMPUTER NETWORKS

- UNIT I INTRODUCTION AND PHYSICAL LAYER
- UNIT Il DATA-LINK LAYER \& MEDIA ACCESS
- UNIT IIL NETWORK LAYER
- UNIT IV TRANSPORT LAYER
- UNIT V application Layer


## USES OF BLOOMS TAXONOMY

- Create assessment
- Plan lessons
- Evaluate the complexity of assignments
- Design Curiculum gap
- Develop onime cuurses
- Self assessmen:


## MAM SCHOOL OF ENGINEERING

## SIRUGANUR TRICHY

## DEPARTMENT OF MECHATRONICS ENGINEERING

ACADEMIC YEAR 2019-2020 ODD SEMESTER

## TEACHER TEACH TEACHERS (TTT) SCHEME

Date: 9.9.2019

| Speaker | : Mrs.Deepika |
| :---: | :---: |
|  | Assistant Professor |
|  | Mechatronics Enginc |
| Staff Attended | : Mr.M.Chandrasekar |
|  | Mr .Ravichandran |
|  | Mr.Tamilarasan |
|  | Mr.Arumagasamy |
|  | Mr.Karthikeyan |
|  | Mr.S.Saravanan |
| Topic | : 5G Technology |
| Venue | : Smart Class |
| Date | : 9.9.2019 |
| Time | : 2.30pm to 3.30pm |




## 5G TECHNOLOGY

Presented by,<br>J.Deepika<br>Assistant Professor<br>Mechatronics Engineering<br>MAM School of Engineering



## What does it offer?



- Worldwide cellular phones




| COMPARISON OF 1GTO 5 G TECHNOLOGIES |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 14. | 18 | Triext | Cr | tor | 3 |
| 15.5.40.0. | 1970/198.4 | 1980/1999 | 1990/2002 | 2000/2010 | Eonlysits |
| 15, 5 | 2kbps | 14-6, kbps | 2 mbps | 200mbps | S.es |
| M2, | Analog cellular | Dizital cellular | (trumblinatrik) bicheas:lap Incknimy | , ixwey | EDram\% |
|  | Mobile telephony | Disul roseatigt uncontian | tategrated kigh quaily aneir viku $s$ dats | INweina ant of henwat ecrice wailative Kover |  |
| 54a: 7- | FDML |  | CDMLA | CDSH | Condes |
|  | Circuit | (mancorls | Pablet exeryt ke ati iblortiue | All packer | Mrser |
|  | PSIN | PSTN | Packet network | Internet | Ethin |
| Ansin: | Horizontal | Horizontal | Horizunta! | Horitontalk Vertical | Fisisic |


| Key concepts |
| :--- |
| - Real wireless world with no more limitations with |
| access \& zone issues |
| - Wearable devices |
| - IPv6, where a visiting care of mobile IP address is |
| assigned according to location \& connected network |
| - One unified global standard |
| - Smart radio |
| - The user can simultaneously be connected with |
| several wireless access technology |
| - Multiple concurrent data transfer path |




## Network Layer



- All mobile networks will use mobile IP
- Each mobile terminal will be FA (Foreign Agent)
- A mobile can be attached to several mobiles or wireless networks at the same time
- The fixed IPv6 will be implemented in the mobile phones
- Separation of network layer into two sub-layers:
(i) Lower network layer (for each interface)
(ii) Upper network layer (for the mobile terminal)

Open Transport Protocol (OTP)


- Wireless network differs from wired network regarding the transport layer
- In all TCP versions the assumption is that lost segments are due to network congestion
- In wireless, the loss is due to higher bit error ratio in the radio interface
- 5 G mobile terminals have transport layer that is possible to be downloaded \& installed - Open Transport Protocol (OTP)
- Transport layer + Session layer = OTP




## Features (Conti...)

- High quality service based on policy to avoid error
- Support virtual private networks
- More attractive \& effective
- Provides subscriber supervision tools for fast action
- Less traffic
- 25 Mbps connectivity speed
- Enhanced \& available connectivity just about the world
- Uploading \& Downloading speed of 5 G touching the peak (up to 1 Gbps)
- Better \& fast solution



## Conclusion

- 3G-Operator Centric,

4G-Service Centric whereas
5G-User Centric

- We have proposed 5 G wireless concept designed as an open platform on different layers
- The new coming 5 G technology will be available in the market at affordable rates, high peak future \& much reliability than preceding technologies


## Applications of 5 G

- Wearable devices with AI (Artificial Intelligence) capabilities
- Pervasive (Global) networks
- Media independent handover
- Radio resource management
- VoIP (Voice over IP) enabled devices
- With $6^{\text {th }}$ sense technology



## MAM SCHOOL OF ENGINEERING

Siruganur, Tiruchirappalli-621 105.
Department of Mechatronics Engineering

## Teacher Teach Teacher (TTT)

Academic year (2019-2020) Even semester
Date: 21.01 .2020
Speaker: Mr.M.Arumugasamy
Assistant Professor-Mechanical Engineering

Staff attended:

1. Mr.M.Chandrasekar
2. Mr.Karthikeyan
3. Ms.M.Suba pradha
4. Mr.S.Ravichandran
5. Mr.S.Saravanan
6. Mrs.P.Sudha

Topic:
Entrepreneurship Development
Venue:

Smart class
Date \& Time:

$$
21^{\text {st }} \text { Jan } 2020 \& \text { 1.30 P.M to 2.30 P.M }
$$

**enclosure: Report


## REPORT

The session was initiated by Mr.M.Arumugasamy Assistant Professor/Mech, the topic for the Lecture is Entrepreneurship Development and discuss about the following topics

- Entrepreneur
- Intrapreneurship
- Employee satisfaction
- Motivation
- Entrepreneur types
- Characteristics of Entrepreneur

The session comes to an end with the explaining the overview of Entrepreneurship Development and its Types.



Entrepreneurship


## Entrepreneurs V. Intrapreneurs

- Entrepreneurs are people that notice opportunites and take the initiative to mobilize resources to make new goods and services.
- Intrapreneurs also notice opportunities and take initiative to mobilize resources, however they work in large companies and contribute to the innovation of the firm
- Intrapreneurs often become entrepreneurs.


## Intrapreneurship

- I.carning organizations encourage intrapreneurship.
- Organizations want to form:
- Frotuct Chaimpions: people who take ownership of a product from cascept to narker.
- Shunhwuth a group of intrapte neurs kept separate from the rest of the organizatioti.
- New Ventare Division: allows a division to act as its own smaller
sompany.
- Rewards for innovation: link innovation by workers to valued moards


## Small Business Owners

- Small business owners are people who own'a major equity stake in a company with fewer than soo employees.
- In 1997 there were 22.56 million small business in the United States.
- $47 \%$ of people are employed by a small business:



## Advantages of a Small Business

- Greater Opportunity to get rich through stack options
- Feel niture important
- Feclavie secure
- Comfor Leval



Disadvantages of a Small Business

- Lower guaranteed pay
- Fewer benefits
- Expected to have many skills
- Too much cohesion
- Hard to move to a big company
- Large fluctuations in income possible




## Key Personal Attributes

- Entrepreneurs are Made, Not Born!
- Many of these key attributes are developed earty in lib. with the lamily environment playing an impartaite olle
- Entreprencurs tend to have had sel/ empleyed poirefts who tend to support and encourage independendt, achievement, and responsibility
- Ilistborns tend to have more entrepreneurislateribused because they recreive more attention, have to forge their own way, thus creating higher self-confidence

Key Personal Attributes (cont.)

- Need for Achievement
- A person's desire cither for excellence or to succeed in competitive situations
- Iligh achievers take responaibility for attaining their goals, set moderately difficule goals, and wast immediate feedback on their performance
- Success is messured In terms of what thoee effortis thave accomplished
- McClelland's research

| Key Personal Attributes (cont.) <br> - Sell-Sacrifice <br> - 「wostisl <br> - No. hing weith having is fiee <br> - Success has a high price, and entrepreneurs have to be valing te sacrifice certain things |  |  |
| :---: | :---: | :---: |

Technical Proficiency

- Many entrepreneurs demonstrate strong technical'skidst
typically bringing some related experience to their buisiness. ventures
- For example, successful car dealers usually haye lod of technical knowledge about selling and servicing automobiles before opening their dealerships
Especially important in the computer industry
- NOT ALWAYS NECESSARY


# MAM SCHOOL OF ENGINEERING 

Siruganur. Tiruchirappalli-621 105 .

## Department of Mechatronics Engineering

## Teacher Teach Teacher (TTT)

Academic year (2019-2020) Odd semester

```
Speaker: Mr.Karthikeyan
Staff attended:
    1. Mr.M.Chandra sekar
    2. Mr.S.Saravanan
    3. Mr.Tamilarasan
    4. Mr.Arumugasamy
    5. Mr.Ravichandran
    6. Mrs.Deepika
```

    Assistant Professor- Mechatronics Engineering
    Topic:
Organic Light Emitting Diode Displays
Venue:
Smart class

Date \& Time:

$$
7^{\text {th }} \text { August } 2019 \text { \& } 1.30 \text { P.M to 2.30 P.M }
$$

**enclosure: Report


PRINCIPAL

## REPORT

The session was initiated by Mr. S.Saravanan Assistant Professor/Mct, the spic for the Lecture is Organic Light Emitting Diode Displays and its Applications and discuss about the following topics

- Organic Electronics
- Organic Solar Cells
- Organic LED
- AMOLED
- OLED Types
- OLED Applications
- OLED Television

The session comes to an end with the explaining the overview of OLED and its Application.


## introduction

For the past forty years norganic sicon and galium arsenide semiconductors, silicon doxid insulators, and metals such as aluminum and copper have been the backbone of the semiconductor industry However, there has been a gowing research eflort in "organic eloctronics" to improve the semiconducting, conducting, and light-emitting properties of organics (polymers, ofigomers) and hybnds (arganic-inorganic composites) through nove synthesis and self assembly lechniques. Performance improvements, coupled with the ability to process these 'active' materiak at low temperatures over large areas on materials such as plastc or paper, may provide unique tectinologes and generate new applicaions and factors to address the growing needs tor pervasive compung and incodccmectiviy
If we roviow the growh of the electronics industry. IIs clear that novaive organic materials have been essental to the unparateded pertor we see today However. the majorties of these dsplays at the cons either used as sacrificial stencils (photoresists) or passive insulators and arganc matenis are ine electronic functioning of a device They do not conduct current to ect take no active role n ine elecronc luncticnitg
The abifty of chemists to optimize the properties of the organic materials described above has The abify of chemisis lo optimize the properties of he a ganic mate However, it is possible in the near future we may reech the limits of performance improvements in silcon devices, magnetic storege, and displars that can be provided at a reasonable cost. As in the past, basic research on material's may provide a path to new product form factors.
So nontradtonal matenals such as conjugeted organio molecules, shorl-chain oligomers, longer chan polymers, and orgenic-inorganic composites are being developed that emit lighi conduct current, and act as semiconductors. The ablity of these materials to transport charge (holes and electrons) die to the n-orbital overlap of neighbcring molecules provides their semiconducing and conducting properties In addation to their electronic and optical properties, many of these thin-film materials possess good mechenical properties (hexibily and toughness) and can be processed at low temperatures using lechniques lamilia to the semiconducting and printing industnes, such as vecuum evaporation, solution casting. Ink-el printing, and stamping These properties could bad in cost information displays on manulacturing could be used to create products such as low- cost informain) lags.
flexible plastic, and bgc for smari cards and radio-liequency idenuicaion ment of organic lightSimilar enhancements in pertormance hove been soan Eastman Kodak in 1987 on evaporated emiting dodes (OLEDs) Phoerng Wivesity in 1990 on solution-processed semiconducting smal molecules and at Cambrige unversy luminous efficiencies of derivatives of these polymers Currently, the highest obseved libs, thus eliminating the need for the backlight matenals exceed that of
The electronic and ootical properties of these "active" organic materials are now suitable for The elecronic and optical propst electronic products that can address the needs for Ightweight some low-periormance, 21 cocstelecy.

## ORGANIC ELECTRONICS

Organic electros or plastic electronics, is a branch of electronics that deals with conductive polymers. plastics, or small molecules It is called 'organic' electronics because the polymers and small molecules are carbon-based, like the molecules of living things. This is as opposed and small molecules are carbon-based, like the molecules of suing as copper a silicon.
The men principally credted for the discovery and development of highly-conductive organic The men princpaly Coder Alan G MacDiarmid, and Hideki Shirakawa, who were jointly pormers the Nobel Prize in Chemistry in 2000 for the 1977 discovery and development of oridized iodine-doped polvacetylene.

## Abstract:

Orgaric lightemilting diodes (OLFDs) operde on the prinaple of corveting eloctricol energy Organic lighteriitting diodes (OLFDs) operate on the prinapio dorveting owchos wayl ight, a phenomenon known as electrou argescencempunds (carton, hydrogen and electroluminescent layer compnsed of a film a aganc compundosert matanal sandwated oxygen, In its simplest between two electrodes. When an eilecric auric layer. Ifht is emited with a color that depends on the particula materiat uned the organic layer, ight is emited win a colel in flat panel deplays they hase sone advartages
 over backlit active-matnx LCO osplays: geater visporiat actualy it up consumes power, the mont response Since only the por of the deas pomer
Based on these advallages. OLEDs have been proposed for a wid range of displyy applications including magnified micru displays wearable head-mourted cmoputers dgtal cameras. personal digital assistants, smart pagers, virtusl reality games, and moble phones an well as medical, automotve, end other industrial applications

## Key Words: <br> - OLED

An organic lightemitting dode (OLED), abo Light Emilting Polimer (L.EP) and
Is Organic Electro-Luminescence (OEL), is any lightemiting dode (LED) whove emissive electroluminescent layer is comprised of a film of organic compounds. The layer usually contains a polymer substance that allows sutasee orgmic compound to be deposited. They ere daposited in rows and colums anto a la catner by simple "printing" process. The resulting matrix of pixels can emit ight of dilerant colors.

- OLED Components

Like on LED, on OLED is a solidstate genimenacter dovico tha is 100 to 500 nanometers thick or about 200 times smalle then a human har. CLEDs can hase ether two layers or three layers dorgnic matersi, in the bite deger

Conclusion: LED is grining immense apolication in day to day OLED is a rintitunzed LED which will be used for extended visual applications
4.3 How do OLEDS Emit Light? manner to LEDs, trough a process called eledrophosphorescence


The process is as followis

1. The battery or power supply of the devico contaning the OLED applies a voltoge across the OLED
2. An eloctrical current fows from the cuthode to the anode trough the organic liperns An electrical current is a llow of electrons)
he cathode gives electrons to the emisave layer of crganic mobculen
The anode removes eloctions from the conductive lisger of apanic moleouley The anode removes elocion
(Ths is the equivalent to ging electron holes to the condective layer.)
3 At the boundary betwicen the emisave and the conductive lajerse obatons -1 ulection holes
When an electron finds an elaction hole, the election
Whergh eviel a the stom thar's missing an eloctron) the form of a photon of light 4 The OLED emits ight



 used in lage advertising sgs, where tey are wishlor af bry the ato the the smit Such arystaline LEDs are not inexpensive and it is wiry dica high-resolution dspleys
high-resolition dspiers


### 4.2 OLED Components








An CLED consis of fel laming patis
thin The enterte mearth th CuED
 Anede trianquarit)

 Conducting layer. Ths beer is mabe of orywid in C.FDr s apyonde

 condicing lyer) that truigor dictrons foon te exv
 inple pietons ahen a curent fows trough te deocn
and cathode determining which pxels get turned on and which puels ent bnghtness of each pixel is proportional to the amount of applied current


PMOLEDs are easy to make but they consume more powe than other types of OLED, maniy To the power needed for the extermal crcuitry PMOLEDs are most efficient for text and dons and are best sited lor small scieens (2- to 3-inch diagonal) such as those you find in cel pons and are best suited ior amals PDAE and MP3 PLAYERS Even with the external arcuitry, passive-matrix OLEDE consume less batiory powar than the LCDs that aurrently power these devces

## Active-matrix OLED (AMOLED):

AMOLEDS hive full byers of cathode, orgaric molhculns and anode, but the anode iay overiows a flinfilm trascitor (TFT) array that forms a matrix The TFT array itself is the arcutry that determines which puels get turned on to form an image


AMOLEDs consume hes power than PMOLEDs because the TFT array requies less power AMOLEDS consume hes power than PMoLED lacaus displays AMOLEDs also heve faster than etarnal arcutry, of they The bes uses for AMOLEDs are computer monitors, largegreen TVS and ebctronic signs or billbards

5 The color of the light depends on the type of organic molecule in the emissve layer Manufacturers place several types of organic films on the same ate dsplays.
6 The intensly or bnghtness of the light depends on the amount of electical current applied the more aurrent, the brighier the light

### 4.4 Making OLEDs:



## Laboratory set up of a high-precision inkjet printer for making polymer OLED

displays
can be done in three ways
can be done in three ways Vecuum depostion or vecuum thermal evaporaton (VTE) - In a vecuin chanber. the organic molecules are gently heated (evaporated) and allowed to condense as tha
filins onto cooled substrates. This process is expensve and ineflicent
Organio vapor phase dopostion (OVFO) - In a bw-pressare. hat-waled roatic
a molouks onlo cowled sibstrate Where they condense moto inn films Using a cance gas increases the effomey and where they condense into
reduces the cost of making OLEDS

Inkjet printing - With inkellechnobgar. OLED s are spraped onlo substates vat he

 displays like 80 -inch TV eresens or alectionic billoards

## OLED TYPES

There are several types of OLEDs

- Passive-matix OLED
- Active matrix OLED
- Trasparent OLED
- Top-emiting OLEO
- Foldablo OLE
- Write oled


### 5.1 Passive and Active Matrix OLEDs: <br> Passive-matrix OLED (PMOLED) <br> PMOLEDs have strips of calhode orgario leners and srips et enocte The ancie sryse ae arranged perpendculer to the calhode strips The intersections of the cahocte and ancle matir


and cathode, determining which prests get turned on and which pixels remain off Again, the bnghtness of each pwel is proportional to the anount of applied current.


PMOLEDs are easy to make, but they consume more power than other types of OLED, mainly due to the power needed for the external circutry. PMOLEDs are most efficient for text and toons and are best suted for smail screens (2-10 3-nich diogonal) such as those you find in cell phones, PDAs and MP3 PLAYERS. Even with the external circuitry, passive-matrix OLEDs consume less battery power than the LCDs that currently power these devices.
Active-matrix OLED (AMOLED):
AMOLEDs have full layers of cathode, organic molecules and anode, but the anode layer overiews a thin fitm transistor (TFT) array that forms a matrix. The TFT array itsell is the circuity that determnes which pixals get tumed on lo form an image


AMOLEDs consume less power than PMOLEDs because the TFT aray requires less power than external circuitry, so they are efficient for large displays. AMOLEDs also have faster refresh rates suitable for video The best uses for AMOLEDs are computer monitors, largescreen TVs and electronic signs or billboards

5 The color of the ligtt depends on the type of orginc molecien in tre enisuen iner Manufacturers place several types of organic lims on the sens O.ED bi mav antr displays
6 The intensty or bugtiness of the igt depends on the anourt de ehatiof ertets appled the more arrent, the bighter the light
4.4 Making OLEDs:


Laboratory set up of a high-precision inkjet printer for making polymer OLTD displays
The biggest pat of manulacturing OLEDs is applying the orpanic livers te the mbedret Thes can be done in three ways.

Vocuum deposition or vacuum thernal evoporation (VTE) - In a vawin chariber the organic molecules are oontly heated (evaporded) and allowed to conderna is the fitms onto cooled substrates This process is expensee and ineffiount
 chamber, a carier gas trmsports evaporated argane inclecules onto cooked ebbetwes. where they condense into fin filis Uwing a cimer gis increases the effoency and reduces the cost of making OLEDS

 OLED manutacturing and abows OLEDs to bo printed orto very tope firss for lapp diplays lies 80 -inch TV screens or electronic biltoards

OLED TYPES
There are several thes of OLEDs.

- Prosive-mittix OUFD
- Active matrix OLED
- Traneparent OLCD
- Topemitting OLED
- Foldibe DEED
- wrie ClED
5.1 Passive and Active Matrix OLEDs:

Pas sive-matrix OLED (PMOLED):
 araypd perpendala to te cathode trice. The intersections of the cotoob nd mode man


## OLED APPLICATIONS

## 11 Current OLED Applications:

Most of them re cellular phones or LiEu techicay is ameady used ilser products use this new technology
Cellular/mobile phones
Cellularimobile phones There are many mobile phones ill these models use an extemal OLED screen with different



硅 The 588 phone from benc-siemens uses a two ofers several mobile phones with an OLEO 262 k colors and $176 \times 220$ pixels. Lo Elolay powered with the new technology. LG's model technology. LG LP4100 has an external deplay powith 262000 colors and a resolution of 176
Vx8300 hes an Organic lightemiting dode dsploy $\times 220$ pixels
$\times 220$ pixels
Other mobile phone manufacturers ine Moir external dsplays

## MP3 players

 MobiBLU stips an mp 3 player that fealures an Lerganic LED dsplay with 262 k colors. The Sony NWpopular Creative Zen Micro has also an ind dsplay. The Zen Skek music player from Creative A 3000 and NW-A 1000 both heve an OLED d splay has a new $1 /$ inch

## an OLED screen. <br> Digital cameras

The Kodak EasyShere LS633 is the world's first dgtal camera with an organic LED dsplay. The Sanyo Xacti HD 1 is a high definition camera that features an OLED dsplay. Other
OIED screen are from Hasselblad (H2D-39 and 503CWD tor example):
cameras with an OLED screen are from Hasselblad (H20-3


Photo Courtesy HowStuffWorks Shopper
Sod
Corporation announced that it was begnning mass p
In September 2004. Sony Corporation announcer inal entertainment handtheids

LCD means liquid orysial display and its lectinobgy is widely used in modem leienson LCD means liquid oysal ispler LCDs are nonarganic and nonemissive devicas thas means screens or computer any form of light. They just pass or bibx tiv badigiting sydem LCDs ase they do not produce any igh source This is caled a backighting systis
ight sourre Tisis calle than OLED displays imily because mo badighting is nessed in OLED displays consume much less power. simply because in Oadigning so wors igtier (ED. displays consume mudntast and are brigtier than LCDS OLED alows abie). This is addition, they have a higher contras and angle is wider (up to 160 dod
and more feribie designs, the viewng ahereas LCDs nees a bacid gri c computer or lievison because they produce there own in the old tradional tectmoiogy used in computar als beam of CRT means cathode ray lube it line an electronc vesuum tube enpoynge wole vewing ongle screens. A cathode ray Ube technobgy are cheap to prodsee and hare cisc but the power electrons Displays wh CRI Bocrobsy arstintensive than produrang Cats bat tive pow Manufacturing of LCDS is males consumplion is lower and the shals CRTs.
eleciromagnos to the cerled tedimiony bit thee are aiso some disadiatiages we nant it
 mention The fletme is imited, eqpecally those of can easily dariage QEFS os compie more cost-miensve


## desplay

### 6.3 Drawbacks:

he aggest ledrical procikm for OIFDS is the limnes likime of the opgenc materass in




 dfiererces in ensig
 The irrusen seaing proesses ane
longevtr di nore lexible dspitigr

Furthermore, XEL-1 can control all the phases of light emission from zero to peak brightness, generating color expression and subleties conventional dsplays camot match

High peak brightness: Faithfully reproduces picture glow
Super Top Emission', a technology unique to Sony and incorporated in its "Organic Panel' has a high aperture ratio which allows for efficient light emission from the organic materials, ealizing high peak brghtiness. This enables "XEL-1" to tatthfully reproduce light flow such a reflections of sun light or camera flashlights through the image reproduced on the display

Exeellent color reproduction: Delivers pure and vivid colors in both dark and brigh images Sony developed its own proprietary organic materials, with bright cobration. In addition, the Sony developed its own propnelay organic mate end the color extracting tectnology within its micro-cavily color filter enable 'XEL-1" to reproduce natural colors beautfully. As a result, the fresh colors of ripe fruit and shades of deep cobatt blue can be stunningly reproduced The "Organic Panel" can also sustain its color reproduction capabiity in scenes of diminished binghtness, enabing 'XEL-1' to fathfully recreate even dark movie scenes using the colors that were orignally intended.
Rapid response time: Smoothly reproduces fast moving images such as sports scenes Since OLED can spontaneously turn the light emitted from the organic material layer on off, OLED is capable of very rapid response times Newly developed OLED dive cicun enable " $X$
naturally

## Low power consumption

aiFD der not recuire a senarate light source due to its ightemiting structure, therefore it OLED does not require a separate ighi scurce dhe means that OLED TVE canoume extromely low can be powered using very low voltagos This meaces The power consumption of 'XEL-1' is as low as 45 W
7.2 Future Applications:

Reper and dien of is procoeding rapidly and may lead to future Resesrch and develoment in the field of OLEDs is procosd billboard-type displays, home and apolications in heads-up displays, automotive dashboards, bisp refresh faster than LCDs - aimost 1,000 office lighting and flenble displays Becouse OLED times faster - a device with an OLED display constantly updated.


Photo courtesy Sony Cornoretion
OLED display for Sony Cie Several companies have ateady bult prototype compuler morncrs and that it had developed a use OLED technology in Mas prototype 40 -inch. OLED-based. ulra-sim TV, the first of its sze

Sony 11 -inch XEL-1 OLED TV:
So actor 2007 . Sony announced that it would be he first b matet with an OLED Niovaion
 The XEL- 1 will be ava
-or chout $\$ 1,700 \mathrm{U} . \mathrm{S}$
 the Sony 11 inch XEL 1 OLED TV.

## Main Features of "XFL-1"

Thinness: Proposes new TV form factor measuring approximately 3 mm thimess (at its thinnest poin!)
As OLFDs are ightemitiog them is nonend for a separate light source such as a backight sytem Sony's "Organic Panel" consists of an organe materia layer of pst sverat hunded syuemnetors thickness, with two extembly thin plass panels algned on eher side of the organic materie thinnest point
High contrast: Reproduces realistic images using exquisite shades of black and flexible High contas rolor tone and gradation
control of color tone and gradation With its light-emitting structure, the OLED display car preventightemissan when (oontrast rato $1,000,000$ 1)


## OLED keyboard

mpany has showed a puotedpe of an OL ED kerptoard The beys are displaped with OLED lectinctogy Thus the whole keyboerd is highly conflgroble The postion, appearance and function of tin kris me math tie in adition the kentond because of its LEDs
the keys can display cons as woll as regular symbols iss posabio to sasoonte beys with nathematical functions. HTM codes or ofter special dincxiers if is sivo possibb bocorfigre gaming keyboard layout for frest person shookers stritigy games or ofser purposes There preconfig

## Windows that light-up at dark:

Windows that fight-up at dark: It is true, this culd be possble with acermal window at day, but af ngit $t \mathrm{can}$ be used as a transparent A wndow could act as a cesource. This vison can reples the boing cid bub in the midde of every roon it an ight resource. This vison can replace the boing better nearly every surfoce can become a iftting sourse t deas not natier it io curved or flat - OLED sheets are fleoble and utra-flat
OLEDs can mimic a natural feeling of ligtt in the dark if urned oft they ane trixsparnint - an ideal precondition for windows it is also imagnable that tibies, appocarch or ofter hemere ae used as a light source
The problem is (as in generst for OLFD) the fert burnout d the blie component Blie is ane of the major colbrs needed to make white ligt, Physort are waskig to moche this mobiem The newspaper of the future might be an OLED duplay that niteshes neth treaking news ind like a regular newspaper, you could fold it up when youte dore reading it and tidk it in pou backpock or briefcase

## MAM SCHOOL OF ENGINEERING

## SIRUGANUR TRICHY

## DEPARTMENT OF MECHATRONICS ENGINEERING

 ACADEMIC YEAR 2019-2020 ODD SEMESTER
## TEACHER TEACH TEACHERS (TTT) SCHEME

```
-Speaker : Mr.S.Saravanan
    Assistant Professor
    Mechatronics Engineering
Staff Attended : Mr.M.Chandrasekar
    Mr.Karthikeyan
    Mr.Tamilarasan
    Mr.Arumugasamy
    Mr.Ravichandran
    Mrs.Deepika
    Topic : Shape Memory Alloy (SMA) and its Applications
Venue : Smart Class
Date : 18.07.2019
Time : 2pm to 3pm
**Enclosure Report
```


## SHAPE MEMORY ALLOY

Presented by, S.Saravanan Assistant Professor Mechatronics Engineering MAM School of Engineering

## ABSTRACT

The aim of this seminar is an introduction to shape memory alloys, the materials that change shape on applying heat. This paper contains a brief history, description of general characteristics of the shape memory alloys and their advantages and limitations. At the end are mentioned groups of most widely used commercial applications.

## CONTENTS

1. Introduction.
2. Brief history.
3. Definition of a shape memory alloy.
4. Types of shape memory effects.
5. Pseudo-elasticity.
6. Advantages and disadvantages.
7. Applications.
8. Conclusion.
9. References.

## 1. INTRODUCTION

Metals are characterized by physical qualities as tensile strength, malleability and conductivity. In the case of shape memory alloys, we can add the anthropomorphic qualities of memory and trainability. Shape memory alloys exhibit what is called the shape memory effect. If such alloys are plastically deformed at one temperature, they will completely recover their original shape on being raised to a higher temperature. In recovering their shape the alloys can produce a displacement or a force as a function of temperature. In many alloys combination of both is possible. We can make metals change shape, change position, pull, compress, expand, bend or turn, with heat as the only activator.
Shape memory alloys have found use in everything from space missions (pathfinder and many more) to floral arrangement (animated butterflies, dragon flies and fairies), from biomedical applications, to actuators for miniature robots and cell phone antennas and even eyeglasses use SMA wires for their extreme flexibility.

## 2. HISTORY

First observations of shape memory behaviour were in 1932 by Olander in his study of "rubber like effect" in samples of gold-cadmium and in 1938 by Greninger and Mooradian in their study of brass alloys (copper-zinc). Many years later (1951) Chang and Read first reported the term "shape recovery". They were also working on gold-cadmium alloys. In 1962 William J. Buehler and his co-workers at the Naval Ordnance Laboratory discovered shape memory effect in an alloy of nickel and titanium. He named it NiTiNOL (for NickelTitanium Naval Ordnance Laboratory).

## 3. DEFINITION OF A SHAPE MEMORY ALLOY

Shape memory alloys are a unique class of metal alloys that can recover apparent permanent strains when they are heated above a certain temperature. The shape memory alloys have two stable phases - the high-temperature phase, austenite and the low-temperature phase, martensite.

[^17]

## 4. TYPES OF SHAPE MEMORY EFFECTS

### 4.1 ONE WAY MEMORY EFFECT

If an alloy, which is in a state of self-accommodated martensite, is deformed by applying mechanical load and then unloaded, remains deformed. If the alloy is then reheated to a temperature above the austenite finish temperature, it recovers original macroscopic shape. This is so called one-way memory effect. During the one-way memory effect internal structural changes take place. When we apply load to the self-accommodated martensite, this structure becomes deformed through variant rearrangement, resulting in a net macroscopic shape change. If the alloy is now reheated to a temperature above the martensitic transformation range the original parent phase microstructure and macroscopic geometry is restored. This is possible because no matter what the post deformation distribution of martensite variants, there is only one reversion pathway to parent phase for each variant. If the alloy is cooled again under martensitic finish temperature, a self-accommodated martensite microstructure is formed and the original shape before deformation is retained. Thus one-way shape memory is achieved.

### 4.2 TWO WAY MEMORY EFFECT

In one-way memory effect there is only one shape "remembered" by the alloy. That is the parent phase shape (so-called hot shape). Shape memory alloys can be processed to remember both hot and cold shapes. They can be cycled between two different shapes without the need of external stress. Two-way shape memory changes rely entirely on microstructural changes during martensitic transformation which occur under the influence of internal stress. Self-accommodation of the martensite microstructure is lost in the two-way effect due to the presence of these internal stresses. Internal stress may be introduced in a number of ways. Usually we talk about "training" of shape memory alloy. Internal stress is usually a result of irreversible defects which can be introduced through cyclic deformation between hot and cold shapes at a temperature above austenite finish temperature.


Figure 3: Starting from martensite (a), adding a reversible deformation for the one-way effect or severe deformation with an irreversible amount for the two-way (b), heating the sample (c) and cooling it again (d).

## 5. PSEUDOELASTICITY OR SUPERELASTIC EFFECT

One of the commercial uses of shape-memory alloy exploits the poesdo-elantic properties of the metal during the high-temperature (austenitic) plase. The frames of reading glasses have been made of shape-memory alloy as they can underge large deformations in their high> temperature state and then instantly revert back to their original slape whes the stress is removed. This is the result of pecudo-clasticity; the manconitic phase is goserated to stressing the metal in the austenitic state and this marfensite phase is capuble of large strains. With the removal of the load, the martensite transforms back into the austenite phase and resumes its original shape.
This allows the metal to be bent, twisted and pulled, before reforming its shape when released. This means the frames of shape-memory alloy glases are claimed to be "nearly indestructible" because it appears that no amount of bending pplited en en $^{\text {it results in }}$ permanent plastic deformation.

## 6. ADVANTAGES AND DISADVANTAGES

Some of the main advantages of shape memory alloys include

- Bio-compatibility
- Diverse Fields of Application
- Good Mechanical Properties (strong, corrosion resistant)

The use of NiTi as a biomaterial has severable possible advantages. Its shape memory property and super elasticity are unique characteristics and totally new in the medical field. The possibility to make self-locking, self expanding and self- compressing thermally activated implants is fascinating. As far as special properties and good bio compatibility are concerned, it is evident that NiTi has a potential to be a clinical success in several applications in future.

There are still some difficulties with shape memory alloys that must be overcome before they can live up to their full potential. These alloys are still relatively expensive to
manufacture and machine compared to other materials such as steel and aluminum. Most SMA's have poor fatigue properties; this means that while under the same loading conditions (i.e. twisting, bending, compressing) a steel component may survive for more than one hundred times more cycles than an SMA element.

## 7. APPLICATIONS

## - AIRCRAFT MANEUVERABILITY

The wire on the bottom of the wing is shortened through the shape memory effect, while the top wire is stretched bending the edge downwards, the opposite occurs when the wing must be bent
 upwards. The shape memory effect is induced in the wires simply by heating them with an electric current.

## - BONE PLATES

Bone plates are surgical tools, which are used to assist in the healing of broken and fractured bones. The breaks are first set and then held in place using bone plates in situations where casts cannot be applied to the injured area. Bone plates are often applied to fractures occurring to facial areas such the nose, jaw or
 eye sockets. Bone plates can be fabricated using shape memory alloys.

## - MINIATURIZED WALKING ROBOT

The implementation of SMA wires coupled with a

simple $D C$ control system can be used to drive small objects without the addition of relatively heavy motors, gears, or drive mechanisms.

- ROBOTIC MUSCLE

Shape memory alloys mimic human muscles and tendons very well. SMA's are strong and compact so that large groups of them can be used for creating a tife-
 like movement unavailable in other systems.

## 8. CONCLUSION

The many uses and applications of shape memory alloys ensure a bright future for these metals. Research is currently carried out at many robotics departments and materials science departments. With the innovative ideas for applications of SMAs and the number of products on the market using SMAs continually growing, advances in the field of shape memory alloys for use in many different fields of study seem very promising.

There are many possible applications for SMAs. Future applications are envisioned to include engines in cars and airplanes and electrical generators utilizing the mechanical energy resulting from the shape transformations. Other possible automotive applications include using SMA springs in engine cooling, carburetor and engine lubrication controls.

## 9. REFERENCES

> "Materials Science and engineering" by William D. Callister, Jr.
> http://smart.tamu.edu
> Shape Memory Applications Inc._Shape Memory Alloys. http://www.sma-inc.com/SMAPaper.html

- Mechanical properties and reactive stresses of Ti-Ni shape memory alloys. N. N. Popov, T. I. Sysoeva, S. D. Prokoshkin, V. F. Lar'kin and I. I. Vedernikova.
simple $D C$ control system can be used to drive small objects without the addition of relatively heavy motors, gears, or drive mechanisms.


## - ROBOTIC MUSCLE

Shape memory alloys mimic human muscles and tendons very well. SMA's are strong and compact so that large groups of them can be used for creating a life-
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## 9. REFERENCES

2. -Materials Scienot and enginotring' by Whiliam D. Callitter, Mr.
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## MAM SCHOOL OF ENGINEERING

Siruganur, Tiruchirappalli-621 105.

## Department of Mechatronics Engineering

## Teacher Teach Teacher (TTT)

Academic year (2019-2020) Even semester
Date: 10.2 .2020

Speaker: Ms.M.Suba pradha
Assistant Professor - Mechatronics Engineering

Staff attended:

1. Mr.M.Chandra sekar
2. Mr.S.Saravanan
3. Mr.S.Ravichandran
4. Mr.M.Arumugasamy
5. Mrs.P.Sudha
6. Mr.Karthikeyan

Topic:
Introduction To IPR and its needs

## Venue:

## Smart class

## Date \& Time:

$10^{\text {th }}$ February 2020 \& 1.30 P.M to 2.30 P.M
**enclosure: Report


## REPORT

The session was initiated by Ms.M.Suba pradha Assistant Professor/Mct, the topic for the Lecture is Introduction To IPR and its needs and discuss about the following topics

- Intellectual property rights
- Infringement
- Patent
- Overview of patenting process using PCT
- Industrial Design
- Trademarks
- Copyrights

The session comes to an end with the explaining the overview of IPR and its needs.



## Example of Trade Secret

- Coca Cola:


The compary pre o st the formula or recipe as a Trade Secret known outy to a few employees, tmody exocurive


## INFRINGEMENT

- Nutrisciences innovation Ltd - New pork based herbal medicine company - registered the name "jeevani" at USPTO.
- Great Earth Inc. (Great Earth), another New York based supplement and vitamin companymarketed an energy drink called "Jeevani Jolt $1000^{*}$ that included the same ingredients as those in the original Jeevank.

Research Commercialization at
Universities through IPR cell

- Evaluate inventions for patenting
s University wo s closely with inventors
a Work with egol experts for obtaining and management of patents
- Identify and negotiate with a commercial partner for-license or collaboration agreements
- Managing relationship
commercial paitners

Novelty - New

The invention shculd rot have been published in India or elsewhere The imvention should not have been in prior public knowledge or publicuse in India.
Exception: display in pu ic eahibition or paper presented before a learned society but with in twelve months, patent application should befiled.


## Criteria for Patentability

An invention must pass through all 3 doors of patentability:


Inventive Step or Non-Obvious

- "Inventive step" means a feature of an invention that involves technical involves advancement as compared having economic significance or both and that makes the invention not obvious to a person skilled in the art.
- The question, is there any inventive step?' arises only if there is novelty.


## NON PATEN TABLE INVENTIONS

Section 3(a)

- Frivolous inventions
- Inventions contrary to well established natural laws
- Examples
> Machine producing mare than $100 \%$ performance.
- A machine alleged to give output without any input.
- Perpetual motion machines


## NON PATENTABLE INVENTIONS

Section 3 (c)

- Mere Discovery of a Scientific Principle or
- formulation of an Abstract Theory or
- discovery of any living thing or
- discovery of non-living substance occurring in nature


## Examples

- Newton's law.
- Darwin's theory.
- Discovery of an animal.
- Discovery of natural gas or a mineral. .

What are not Inventions?.....

 acietien und weovec und ensemtiaid, bidelefical process for medelination or prippupation of animets but other than
(9. Ou uec and Ben workties of pionts
$\sec 26 \pi$

## Provisional Specification

- Should contain title, Problem in the art and nature of the invention with probable solution.
- To claim the priority date of the invention
- Need not contain claims, drawings.
- Complete specification within 12 months


## Complete Specification

- Title of the invention
- Field of Invention
- Uses of the invention
- Prior art
- Drawbacks of the prior art
- Comparison of prior art and the invention
- Summary of the invention
- Drawings
- Detailed description of the invention
- Claims



## INDUSTRIAL DESIGNS

An Industrial design is that aspect of a useful article, which is ornamental or aesthetic. Two-dimensional features like patterns, lines, colors etc, and threedimensional features like shape, surface of the article etc.
e.g. shape of a handle or body portion of a pressure cooker
A Design must be new or original and industrially reproducible in order to become eligible for protection under Industrial designs law. Protection of Industrial designs is territorial and there is only civil remedy avaifable aqainst its infringement

## WHAT IS COPYRIGHT ?

- A Copynght is a protection offered to the works created by the authors of literary,


## Literary work includes

- Literature works expressed in print or writing utilizing notations.
- Books / E-Books
- New editions of books (if substantial change)
- Novels
- Short stories
- Single poem or book of poems
- Song lyrics
- Concept note with adequate details
- Letters
- Lectures, Sermon and Speeches in writing, print and digital format.


## VALUVATION OF IPR

- MERCEDES BENZ - S CLASS - 340

IP Duration - Term of Protection

- Patents -20 years
- GI/ Trademarks - 10 years + renewals
- Copirights in puplisted ititray dra

- Broadcast reproduction right- 25 years from the beginning of
- Performers right- 25 years from the beginning of the calendar
year next following the year in which the perfortance is made.
year next following the yar- 5
- Industrial designs- 10 years+ renewal permitted once for 5
and know how collectively *proprietary
- Trade-secrets and know how eollectively proct provisions technology breach of trust.
- JET AIRIVAYS - 33 Crores worth of IPRs

THANK YOU...

M.A.M. SCHOOL OF ENGINEERING SIRUGANUR, TRICHY-621105
(Approved by AICTE, New Delhi and Affiliated to Anna University, Chennai)
(An ISO 9001: 2008 Certified Institution) (ACCREDITED BY NAAC)

# DEPARTMENT OF MECHATRONICS ENGINEERING 

WEBINAR<br>on<br>"MICRO FABRICATION TECHNIQUES FOR MEMS"

### 26.05.2020



# M.A.M. SCHOOL OF ENGINEERING (Accredited by NAAC) <br> (Approved by AICTE, New Delhil Affiliated to Anna University) Siruganur, Trichy - 621105 

Department of Mechatronics Organises

## Webinar on

Micro Fabrication Techinques for MEMS

## Resource person



## Dr. L. Sujatha,

Head, Centre of Excellence in MEMS \& Microluidics Rajalakshmi Engineering College, Chennai.

Date: 26/05/2020, Time: 11.00am.

## For Registration Visit www.mamse.in

## Resource Person Profile



Dr. L. Sujatha, Head, Centre of Excellence in MEMS \& Microfluidics (CEMM) and Professor in the Department of Electronics \& Communication Engineering, Rajalakshmi Engineering College (REC), Chennai has 30 years of experience in teaching and research. Graduating with an A.M.I.E. in Electronics \& Communication Engineering from Institution of Engineers (INDIA) in 1991, she obtained her M.E. (Applied Electronics) from Bharathiar University in 1996. She has done her PhD and Post-Doctoral Research in the field of Micro Electro Mechanical Systems (MEMS) at Indian Institute of Technology Madras. She is a recognized supervisor under Anna University and guided 3 research scholars for their PhD degree. She has published two book chapters, 40 journal papers in refereed international journals, more than 60 International Conferences. She had received a "Best Woman Engineer" award from Pondicherry Enginecring College in the year 2007 and received "Dr. A.P.J. Abdul Kalam Award for Innovative Research" by Society for Enginecring Education Enrichment (SEEE) in the year 2017. She is a Life Member of various technical societies such as IEI, ISSS, ISTE, IETE and SEEE. She had visited Singapore, European Countries and USA for presenting her research works. She has established a "Centre of Excellence in MEMS \& Microfluidics" at Rajalakshmi Engineering College with sophisticated equipment and Clean Room facilities for in-house fabrication of microdevices. She fabricated many Micro Devices such as Micro Tweezers, Micro-heater, MEMS Gyroscope, Triaxes Accelerometer, Digital Microfluidics etc. She has successfully completed 10 sponsored projects and received funding of Rs 7 Crores from various funding agencies such aş DRDO, DST, AERB etc. She is very passionate on research activities on microfabrication technologies and developing chemical and biosensors.

## M.A.M SCHOOL OF ENGINEERING

Singenur, Tridyy-21105

## Department of Mechatronics Engineering

## Webinar on "Miero Fabrication techniques for MEMS"

 ( $26^{\mathrm{th}}$ MAY, 2020)Department of Mechatronics Engineering had the privilege of having webinar with Dr. L. Sujatha, Head - Centre of Excellence in MEMS \& Micro fluidics, Rajalakshmi Engineering College, Chennai. on the topic of "Micro Fabrication techniques for MEMS".

The invitation for this program by the designer team of MAMSE and distributed through face book. The registration form for this program has created by Google form and published in our college website on $24^{\text {th }}$ May 2020.

The link for the registration:
https://docs.google.com/forms/d/e/IFAIpQL SeavDtiS.JmEbOqoD_LiquFQHW mbsOElhyX20igCYA4v-S4LpA view form

Totally 113 participant has register for this webinar. The session is started sharply by 11 am from welcome address and introduction given by Dr.Punitha, Professor, Mechatronies Engineering Department. After that the resource person starts the lecture, with the introduction of MEMS. Later she gave the lecture with Different types of Fabrication methods and its process and etc. The event ended with vote of thanks given by P.Sudha Head of the Department, Mechatronics Engineering Department. Also the feedback for the participant was collected through Google form.

The link for the Feedback:
https://docs.google.com/forms/d/e/IFAlpQLSdB5SiXHH2Dr7YgEZigi-u2IPvTX9cJlyC-cHfjUdi y5CwVA/viewform

Program Co-ordinator

| SI No | MAM SCHOOL OF ENGINEERING <br> DEPARTMENT OF MECHATRONICS <br> the Webinar "Micro Fabrication techniques for MEMS" |  |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Name | Institution | Department | At Present | Year | Email ID | What about the session? |
| SINO | 2020/05/26 11:48:06 AM GMT +5:30 | Saravanan S | MAM School of Engineering | Mechatronics | Faculty | Not applicable | saravananknm(cgmail com | Excellent |
| 1 | 2020/05/26 11:48:32 AM GMT+5:30 | Copi K | Periyar Centenary Polytechnic college | Mechanical | Faculty | Not applicable | gopiindian555@gmail.com | Good |
| 2 |  |  |  |  | Student | II | satheshkumar4444@gmail com | Excellent |
| 3 | 2020/05/26 11:50:02 AM GMT + 5:30 | A satheesh kumar | Mam school of engineerin | Mechatronics | Studen |  |  |  |
| 4 | 2020/05/26 11:50:03 AM GMT+5:30 | Vaitheeswari. V | Mam school of engineering | Mechatronics | Student | IV | vaitheeswarimechatronics@gmail com | Excellent |
| 5 | 2020/05/26 11:52:12 AM GMT+5:30 | P. Palanisamy | M. A. M. SCHOOL OF ENGINEERING | Aeronautical | Student | III | Palaniaero35@gmail com | Excellent |
| 6 | 2020/05/26 11:52:43 AM GMT+5:30 | K KARTHIKEYAN | M A.M SCHOOL OF ENGINEERING | ECE | Faculty | Not applicable | karthikcacet121@gmail com | Excellent |
| 7 | 2020/05/26 11:54:50 AM GMT+5:30 | R Nirmal | Trichy engineering college | Electrical and Electronics | Faculty | Not applicable | nirmalpse@gmail.com | Good |
| 8 | 2020/05/26 11:54:52 AM GMT+5.30 | VIVEKNIJANTHAN L | Periyar Centenary Polytechnic College, Vallam, Thanjavur | Department of Mechanical Engineering | Faculty | Not applicable | vknijanthan96@gmail.com | Excellent |
| 9 | 2020/05/26 12:03:04 PM GMT+5:30 | N. Timple Rosni Augustina | M.A. M school of engineering | CSE | Student | 1 | timplerosni@gmail.com | Excelient |
| 10 | 2020/05/26 12:05:57 PM GMT+5:30 | S. SRIRAM NIVAS | M. A M SCHOOL OF ENGINEERING | MECHATRONICS | Student | 1 | sriramnivas1104@gmail.com | Excellent |
| 11 | 2020/05/26 12:06:48 PM GMT+5:30 | Saravana Kumar K | Periyar centenary polytechnic college | Mechanical engineering | Faculty | Not applicable | saran07mech@gmail.com | Good |
| 12 | 2020/05/26 12:07:39 PM GMT+5:30 | RAJESHKUMAR | MAM SCHOOL OF ENGINEERING | CSE | Faculty | Not applicable | grkresearch@gmail.com | Excellent |
| 13 | 2020/05/26 12:19:31 PM GMT+5:30 | P. GRANAF | MAM SCHOOL OF ENGINEERING | CSE | Student | 1 | granafpaul@gmail.com | Excellent |
| 14 | 2020/05/26 12:45:21 PM GMT+5:30 | Elangathir | Mam school of engineering | Mechatronics | Student | IV | elangathirmamse@gmail.com | Average |
| 15 | 2020/05/26 12:54:10 PM GMT+5:30 | Dr. LILLY FLORENCE. P | M. A. M. School of Engineering | Chemistry | Faculty | Not applicable | mamsconlineclasses@gmail.com | Good |
| 16 | 2020/05/26 12:55:49 PM GMT +5:30 | MUTHUKUMARAN D | Periyar Centenary Polytechnic college,yallam | Mechanical engineering | Faculty | Not applicable | kumaranmuthu00@gmail.com | Excellent |
| 17 | 2020/05/26 1:02:36 PM GMT+5:30 | Sumathi Sivakumar | Mam school of engineering | Cse | Faculty | Not applicable | dsumihari@gmail.com | Good |
| 18 | 2020/05/26 1:08:28 PM GMT+5:30 | SUDHA P | M. A.M SCHOOL OF ENGINEERING | MECHATRONICS ENGINEERING | Faculty | Not applicable | suha 1906@gmail com | Excellent |
| 19 | 2020/05/26 1:25:39 PM GMT+5:30 | R. Arun Kumar | M. A. M school of engineering | Mechatronics | Student | IV | arunselvi141@gmail.com | Good |
| 20 | 2020/05/26 2:44:37 PM GMT+5:30 | Jenith Kumar.B | M.A.M School of Engineering | Mechatronics | Student | II | jenithbala07@gmail com | Excellent |
| 21 | 2020/05:26 3:25:59 PM GMT+5:30 | LALUPRASHANTH-A | Mam school of engineering | Mechatronics | Student | II | aplalu2001@gmail.com | Excellent |
| 22 | 2020/05/26 3:51:38 PM GMT +5:30 | Prasanth.P | M. A.M school of engineering | MECHATRONICS | Student | IV | josephprasanth1999@gmail com | Excellent |
| 23. | 202005/26 5: 15:58 PM GMT+5:30 | M SUBA PRADHA | M A.M SCHOOL OF ENGINEERING | ECE | Faculty | Not applicable | cool.pradha@gmail.com | Excellemt |
| 24 | 2020/05/26 5:36:04 PM GMT +5:30 | T. Varshini | M. A M. School of engineering | Mechatronics | Student | III- | Varshinithapasu@gmail.com | Good |
| 25 | 2020/05/26.9.42:46 PM GMT +5:30 | PMANOJ KUMAR | MAM-SCHOOL OF ENGINEERING | MECHATRONICS | Student |  | msmano3786@gmail com | Excellemt |



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|  | 44 | 202005/25 953.23 PM GMT 5130 | Beteth Mary 5 | MAM School of Engineering. Trichy | Physics | Faculty | Assistant Professor |  | Female | berbethmaryegrail com | 976emizs |
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|  | so | 20200582612 S5 10 AM GMT +5 30 | Abinas Kumar | MAMSE | Mechatronics | Student | Not applicable | Iv | Male | abinax9199Cegmail com | тมว1วบร9 |
|  | 51 | 20200526 S 53.26 AM GMT 53.30 | vignesh waran | MAM SCHOOL OF ENGINEERING | MECHANICAL | Student | Not applicable | II | Male | nw602396ernail com | wowzmosiz |
|  | 52 | 202005266 2741 AM GMT+530 | K Kaviy | M A M SCholl of engineering | CSE | Student | Not applicable | 1 | Female | an29842300gmail com | 9150860346 |
|  | 33 | 202005 206532.33 AM GMT +530 | B Bharathi mia | Mam school of engineering | Mechanical | Student | Profesor | II | Male | blaratia720018ymail com | 96209poct |
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Total No of Registrations : 113

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## M.A.M SCHOOL OF ENGINEERING

## Department of Mechatronics Engineering

# Webinar on "Arduino with Thinkercad Simulator" 1st June 2020. 

RINCIPAL

# M.A.M. SCHOOL OF ENGINEERING 

(Accre dited by NAAC)
(Approved by AICTE, New Delhil Affiliated to Anna University)
Siruganur, Trichy - 621105
Department of Mechatronics Engineering Organises

Webinar on
Arduino with Tinkercad Simulator

## Resource person



## Prof. Kanagaraj Venusamy/r

Industry and Academic Expert,
Al musanna College of Technology, Sultanate of Oman, Muscat.

Date: 01/06/2020, Time: $3.00 \mathrm{pm}-4.00 \mathrm{pm}$.

## For Registration Visit www.mamse.in

# M.A.M. SCHOOL OF ENGINEERING (Accredited by naAC) 

Approved by AICTE, New Delhi | Affiliated to Anna University Siruganur, Tiruchirapalli - 6211

## Department of Mechatronics Engineering Organises

## Webinar on

## Arduino withTinkercad Simulator

## Resourse Person



Mr. Kanagaraj Venusamy, B.E., M.E., M.B.A., (P.hD) Industry and Academic Expert, Al musanna College of Technology, Sultanate of Oman, Muscat.
oin us on: Tuesday, Znd June, 2020
Fime: 3.00 p.m to 4.00 pm , IST
For Registration Visit : www.mamse.in


E-Certificate will be provided to all participants

## CURRICULUM VITAE

## Personal Profile

Name: Kanagaraj Venusamy
Nationality: Indian
Date of Birth: 19.03.1982

Passport Number: L2303315

Languages speak: Tamil, Telungu \& English

Communication Address:

> Al Musanna College of Technology.
> Directorate of Technological Education,
> Sultanate of Oman-Muscat
> E.Mail :rajkanagaraj1983@gmail.com
> Mobile: 0096891327801

## Objective

To merge my enthusiasm and talent for learning and teaching with students in order to develop professional skills and attitudes.

## Academic Qualification

PhD in Management (Pursuing under Bharthidhasan University)
M.E in Mechatronics Engineering (Rajas college of Technology)
M.B.A in Production (Manonmanium sundaranar University)-May 2011.
B.E. in Electronics \& Communication Engineering from Srinivasa Institute of Engg.

Technology, Tamil Nadu. (Anna University, Chennai), April 2005.
Diploma in Project Planning Management from CADD CENTRE, Dec 2009.

Area of Interest
\& INDUSTRAIL AUTOMATION (PLC)
\& MATLAB/SIMULINK
\& ARDUINO
f VEXROBOTICS
\& ENTREPRENEURSHIP
\& INDUSTRY-INSTITUTION INTERACTION CENTRE ACTIVITY

I hereby inform you that all the statements are made above true the best of my knowledge and belief. Kanagaraj V

## Proposal for online webinar

Title: Introduction to Arduino and Programming
How. Using Real Time open Source Software Tinkercad
Organizer:
speaker/Presenter: Kanagaraj Venusamy B.E, M.B.A, M.E, (Ph.D)
When: yet to plan
Where: Online Webinar
Target Audience: Computer science and Electronics discipline students and faculty but globally Arduino is used by grade4 level to any discipline people.
Interdisciplinary Arduino: Programming, Electronics, Mechanical, hobbyist...


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| Vivekkumarp1p2p3＠grnal．com | 8809250956 |
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| sauravraiput703＠gmailcom | 9998180703 |
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| balisuman2509mal com | 7082071949 |
| k．viayakumar201＠gmailcom | 9159582190 |
| madhumurugesan12＠gmal com | 9000255309 |
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## M.A.M School of Engineering

Siruganur - 621105
List of Participants who attended the webinar on "Arduino with Thinkercad"
Meeting ID: 86122194266
Topic: MAM-Arduino Interface with Tinkercad
User Email rajkanagaraj1083@gmall com
Duration (Minutes) 92
Start Time 06/01/2020 14.4048
End Time: 06/01/2020 16 12:17

|  |  | Total Duration (Mirutes) |
| :---: | :---: | :---: |
| Name (Original Name) | User Lmail | 88 |
| chandrasekar M | mchandrasekar1983@gmail.com | 43 |
| A.p. Lalu |  | 73 |
| Rosy kanna |  | 20 |
| vivo 1713 |  | 72 |
| Sumathi Sivakumar | dsumihari@gmail.com | 6 |
| Akshay Verma 1903220 E2 | vermaakshay80@gmail.com | 84 |
| Senthamarai Kannan | ersenthamarai@gmail.com | 2 |
| Nitish kumar(1902459) |  | 4 |
| P. Palanisamy |  | 78 |
| Kvilaya Kumar |  | 59 |
| RAVICHANDRANS | ravi.thuraiyur0791@gmail.com | 57 |
| umarani |  | 8 |
| A.karthick Kumar | kumar.akarthick783@gmail.com | 92 |
| Kanagaraj Venusamy | rajkanagaraj1983@gmail.com | 3 |
| Vidhyaraj |  | 79 |
| Sumit Taggarh | taggarhsumit00@gmail.com | 14 |
| Priyansh Kumar 1902474 | kumarpriyansh620@gmail.com | 5 |
| Yukta Wadhwa 1902587 E1 |  | 75 |
| Karthik Keyan | karthikcacet121@gmail.com | 2 |
| Parul Sinha |  | 5 |
| vivo 1906 |  | 61 |
| Punitha a | sweetpunitha@gmaii.com | 51 |
| Rajeshkumar Gunasekaran |  | 57 |
| Abirami25 | sbishvender28@gmail.com | 61 |
| Bishvender 1903226 E2 | sbishvender28@gmail.com | 80 |
| Yogeshwaran N |  | 71 |
| Selvakumari M |  | 45 |
| Training and Placement MAMSE | mamseplacement@gmail.com | 16 |
| ArockiarajS | kerthajoses | 27 |
| S.Deepa | jenithbala07@gmail.com | 82 |
| Jenith Balu | saravananknm@gmail.com | 82 |
| Kaviya |  | 14 |
| Navraj Singh 1903238 | navrajsingh2300@gmail.com | 3 |
| Sudha P | suha1906@gmail.com | 53 |
| Ahmed Faisal | ahmedfaisai4171@gmail.com | 62 |
| Yukta Wadhwa 1902587 E1 | yuktawadhwa23@gmail.com | 14 |
| Sowndarya |  | 58 |
| Dimple V. Paul |  | 31 |
| Abdul Latiff S | lamuya.tnj@gmail.com | 36 |
| ARASHDEEP RIAT 1903222 | arashriat3151@gmail.com | 4 |
| Hussain |  | 28 |
| Kavitha P | kaviakshya@gmail.com | 46 |
| Sounthar Pandian |  | 2 |
| Ajay Arun | ajayzion875@gmail.com | 77 |
| Mohit_ Dagar(1902438) | mohitdager4567@gmail.com | 78 |
| V |  | 80 |


| Wiely yodav 1902584 El | vivek 734706 egmail com |  |  |
| :---: | :---: | :---: | :---: |
| xchandra Selaran | kchandrusekaran $1984 \mathrm{Cgmal}^{\text {com }}$ | 52 |  |
| Ksathish Kumar | iski2mohan egmalicom | 33 |  |
| BALAII VISWANATHAN | viswasanitegmail.com | 53 |  |
| B.blbo. | monsterbubloo0egmail.com | 14 |  |
| wo |  | 35 |  |
| Or.P. Lily Florence | mamseonlineclasses @gmalicom | 22 |  |
| SanjaiR | a | 78 |  |
| 8. 017 pl kumar |  | 69 |  |
| Tirtha 1902568 | tistyles555@gmall.com | 71 |  |
| VIVEXNUANTHANL | vinnijanthan96@gmail.com | 1 |  |
| Jeba Priyadarshini J | jebachristinal1995@gmal.com | 41 |  |
| Sudeep das 1903246 E2 | sudeep1098@gmalicom | 76 |  |
| Purushothaman $G$ | eeemamseegmail.com | 39 |  |
| 1flCCOpsfih4xFOSK_901SAAAAAB |  | 45 |  |
| Ryb2lkU2hhcmVfNTgX |  | 11 |  |
| Emmanuel paulraja |  |  |  |
| Manisha Manisha |  | 34 |  |
| Vibhuti Goyal 1902573 E1 | manishamuthu199@gmail.com | 76 |  |
| Murugavalli S | vallisangilimuthu2012@gmail.com | 45 |  |
| Dhanalakshmi D | dhanalakshmi28031996@gmail.com | 60 |  |
| Suba Pradha | cool.pradha@gmail.com | 88 |  |
| S.kulanthaivel |  | 8 |  |
| Pankaj 1902462 |  | 8 |  |
| Ramanathan R | srnathan2000@gmail.com | 24 |  |
| Kutty Vasanth8 | kutty5645@gmail.com | 59 |  |
| Ranjithkumar P | ranjiith@gmail.com | 57 |  |
| Divya |  | 6 |  |
| A.Bhuvaneswari |  | 36 | thr |
| Asraf Banu | asraf71073@gmail.com | 7 |  |
| T. Varshini |  | 33 |  |
| Navya (1902448) | navyasharma307@gmail.com | 25 |  |
| Manoj Kumar | msmano3786@gmail.com | 48 |  |
| P. jackvin jandro |  | 17 |  |
| Yukesh |  | 5 |  |
| Akshay Verma 1903220 E2 |  | 78 |  |
| Ithyas |  | 67 |  |
| S Valarmathy | valarmathysr@gmail.com | 5 |  |

Total No. of Participants: 84

# M.A.M SCHOOL OF ENGINEERING <br> Siruganur, Tichy-621105 

## Department of Mechatronics Engineering

Webinar on "Arduino with Thinkercad Simulator" (15t June, 2020)

Department of Mechantronics Engineering had the privilege of having webinar with Mr. V. Kanagaraj, Industry and Academic Expert, Al Musanna College of Technology, Sultanate of Omen, Muscat on the topic of "Arduino with Thinkercad Simulator".

The invitation for this program by the designer team of MAMSE and distributed through face book. The registration form for this program has created by Google form and published in our college website on $30^{\text {th }}$ May 2020.

```
The link for the registration:
https://docs.google.com/forms/d/e/1FAIpQLSdgIAqikZgN03dtyuvCSXfjBBqa gLDDvogaUYV7AFCE8BBGcg/viewform
```

Totally 157 participant has register for this webinar. The session is started sharply by 3.00 pm from welcome address and introduction given by M.Chandrasekar, . Assistant Professor, Mechatronics Engineering Department. After that the resource person starts the lecture with the introduction of Thinkercad simulator. Later he given the lecture with demonstration of how to connect LED with Arduino, servomotor interface with Arduino and etc. The event ended with vote of thanks given by M.Suba pradha, Assistant Professor, Mechatronics Department. Also the feedback for the participant was collected through Google form.

The link for the Feedback:
https://docs.google.com/forms/d/e/1FAIpQLSde7smlYTLhkKYPKlgmltghjgc $\mathrm{ZOpSr}-\mathrm{H} 0 \mathrm{nYb} 10 \mathrm{Dxlepr} 16 y \mathrm{Q} /$ viewform

Program Co-ordinator
HOD

Feedback Reprot for a Webinar - Arduino with Thinkercad

| Institution | Department | At Present |
| :---: | :---: | :---: |
| Chandigarh group of colleges landran | ECE | Student |
| MAM SCHOOL OF ENGINEERING | MECHATRONICS | Student |
| CEC LANDRAN | B. TECH ECE | Student |
| CHANDIGARH ENGINEERING COLLEGE | ECE | Student |
| MAM school of engineering | Mechatronics | Student |
| M.A.M School of Engineering | Mechatronics | Student |
| M.A.M School of engineering | Mathematics | Faculty |
| MAM School of Engineering | CSE | Faculty |
| Mepco Schlenk Engineering College, Sivakasi. | EEE | Faculty |
| Chandigarh Engineering College | ECE | Student |
| Er.Perumal Manimekalai Polytechnic College | ECE | Faculty |
| Chandigarh Engineering College | ECE | Student |
| Er. Perumal Manimekalai College of Engineering | Mechatronics | Student |
| CEC LANDRAN | B.TECH ECE | Student |
| M. A. M. School of Engineering | Chemistry | Faculty |
| MAM school of engineering | cse | Student |
| M. A. M. SCHOOL OF ENGINEERING | MECHATRONICS | Student |
| CHANDIGARH ENGINEERING COLLEGE, LANDRAN | ECE | Student |
| Nehru memorial collage | Bsc.Chemistry | Student |
| M.A.M School of Engineering | ECE | Faculty |


| Email io | What about <br> the session? |
| :--- | :--- |
| sudeep1098@gmail.com | Excellent |
| msmano3786@gmail.com | Excellent |
| mohitdager4567@gmail.com | Excellent |
| vermaakshay80@gmail.com | Excellent |
| ajayzion875@gmail.com | Good |
| jenithbala07@gmail.com | Excellent |
| krosy.kanna@gmail.com | Excellent |
| grkresearch@gmail.com | Excellent |
| keerthanajose4@gmail.com | Excellent |
| vivek734706@gmail.com | Excellent |
| selvakumari.sundaram@gmail.com | Excellent |
| sumittaggarh786st@gmail.com | Good |
| dilipboopathi15@gmail.com | Excellent |
| Pankajdeswal146@gmail.com | Good |
| mamseonlineclasses@gmail.com | Good |
| deepakarambayam@gnail.com | Good |
| k.vilayakumar0201@gmail.com | Excellent |
| Arashriat3151@gmail.com | Excellent |
| Gowtham2603msd@gmall.com | Good |
|  | karthikcacet121@gmail.com |


| 21 | 20200001 +11:19 Pa gemtes 3 | yor dmple vipaut | ONYANPRASSARAK MANDAL'S COLLIGE AND RESEARCH CENTRE (GOA UNIVERSITY) | COMPUTER SCIENCE | Faculty | dimplevpaui@gmail.com | Excellent |
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| 22 | Fezo/0601 +11:19 PM GMT 533 | Saravanan S | MAM School of Engineering | Mechatronics | Faculty | saravananknm@gmallicom | Excellient |
| 23 | 20200600 2-1120 PM GMT 5.34 | aboul Latif' S | MaM School of Engineering | Mechatronics | Student | lamuya.tnjegmailcom | Excellent |
| 24 | 202008/01 4-175s PM GMT+5:39 | rogeshwaran N | Er. Perumal Manimekalai College of Engineering, Hosur. | BE.Mechatronics | Student | nyogeshwaransgmail.com | Sxcellent |
| 25 | 2000/06/01 $418: 29$ PM GMT+5395 | SUBA PRADHA M | M.A.M SCHOOL OF ENGINEERING | ECE | Faculty | cool.pradha@gmall.com | Excellent |
| 26 | 202005/01 2:1854 PM GMT+5:34, | 2. Arun Kumar | M. A. M school of engineering | Mechatronics | Student | arunselvi141@gmallcom | Good |
| 27 | 2000/06/01 22059 PM GMT+5:39\% | vatheeswariV | Mam school of engineering | Mechatronics | Student | vaitheeswarimechatronicsegmall com | Excellent |
| 28 | 2020/06/01 +22:36 PM GMT+5:34, | MUGEST KUMAR. 8 | MAM SCHOOL OF ENGINEERING | MECHATRTONICS | Student | plsmugesh144@gmallcom | Good |
| 23 | 2020/05/01 4:24:15 PM GMT+5:3/3 | Sanizal | M.A.M SCHOOL OF ENGINEERING | MECHATRONICS | Student | Sanjai1332000segmall com | Good |
| 30 | 2020/06/01 4:26:41 PM GMT+5:34 | Pantaj | Chandigarh group of college | ECE | Student | Pankajdeswal146@gmall com | Excellent |
| 31 | 2020/05/01 4:28:22 PM GMT+5:36 | S praveenkumar | MAM school of engineering | Mechatronics | Student | monsterbubloo0egmall.com | Good |
| 32 | 2020/05/01 4.38:37 PM GMT+5.30 | (RAVICHANDRANS | M A M School of Engineering Trichy | Mechanical Engineering | Faculty | ravi.thuralyur0791egmalicom | Good |
| 33 | 2020/06/01 4:38:53 PMM GMT +5:36 | SENTHAMARAI KANNAN - | M.A.M. SCHOOL OF ENGINEERING | ECE | Faculty | ersenthamaraiegmall.com | Excellent |
| 34 | 2020/05/01 4:41:24 PM GMT+5:30 | M. R. MOHAMMED HUSS; | MAM SCHOOL OF ENGNEERING | Mechatronics | Student | msdhussain514655@gmall com | Good |
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| 37 | 2020/06/01 5:45:25 PM GMT+5:39 | d. Varshini | M. A. M. School of engineering | Mechatronics | Student | Varshinithapasuegmail.com | Good |
| 38 | 2020/06/01 7:37:54 PM GMT+5:30 | Elangathir | Mam school of engineering | Mechatronics | Student | Clangathirmamseegmail com | Good |
| 39 | 2020/06/01 8:32:53 PM GMT+5:39 | M. Manisha | MAM school of engineering | Mechatronics | Student | manishamuthu199@gmail com | Good |
| 40 | 2020/06/01 9:34:14 PM GMT+5:30 | Prasanth. P | M.A.M school of engineering | Mechatronics | Student | josephprasanth1999@gmail.com | Excellent |

# M.A.M. SCHOOL OF ENGINEERING <br> SIRUGANUR, TRICHY-621105 

(Approved by AICTE, New Delhi and Affiliated to Anna University, Chennai) (An ISO 9001: 2008 Certified Institution) (ACCREDITED BY NAAC)

# DEPARTMENT OF MECHATRONICS ENGINEERING 

## WEBINAR

## on

"PRINCIPLES OF ROBOTICS AND ITS APPLICATIONS"

### 18.05.2020



Siruganur, Trichy 621105
Department of Mechatronics Engineering Organises

## Webinaron

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# DraSheejaVFrancis M.S. Find 

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Department of Biomedical Engincerine Jerusalem College of Enginecting Chennal.


## For Registration Visit www.mamse.in

## ACADEMIC PROFILE

Dr. SHEEJA V FRANCIS B.E., M.S.,Ph.D.
Professor \& Head
Department of Biomedical Engineering
Jerusalem College of Enginecring
Pallikaranai, Chennai- 600100
Mobile No : + 919941218230
E mail : sheejavf@gmail.com


Dr. Sheeja V Francis, who is currently, Prof \& Head, Dept of Biomedical Enginceringe Jerusalem College of Engineering, Chennai started her career as a lecturer in Electronics * Communication Engineering, after acquiring her B.E. Degreein 1995. With great passion towards the teaching profession, she has been mentoring and educating hundreds of future engineers over the last 25 years at UG and PG levels. Of the numerous academic courses handled, Electronic Circuits, Signals \& Systems, Digital Signal Processing, Bio Signal Processing, Digital Image Processing.Medical Image Processing and Roboticsare her forte. She has constantly upgraded and shared her knowledge in these areas through several seminars, workshops and NPTEL Courses. She has been constantly motivating students to further their knowledge beyond text books and to find solutions to needs of the society, through innovative projects. She has guided student projects every year leading to conference presentations.

She firmly believes that 'Today's research is the key to tomorrow's technologieal advancements'. Being attracted by the immense contributions made by engineers to the field of health and medicine, she pursued research at the Centre for Medical Electronics, Anna University, Chennai, where she acquired her M.S and Ph.D degrees. She has explored the possibilities of using non-ionising radiations in diagnostics instead of the bio-hazardous X-rays currently in use. She has extensively studied the seope of using Near Infra red light for detecting hematomas in the brain and thermal imaging for deteeting breast cancer, which presently use CT scan and Mammography respectively. She has presented her works at several National and International Seminars \& Conferences and published several research papers in reputed International Journals.

She serves as a member of review panels of reputed international journals with leading publichers such as Elsevier, Springer, Taylor \& Francis, etc.,. She has received the 'Award of Recognition for Outstanding Contribution in Reviewing' from Elsevier Journals, The Netherlands in May 2015 She has contributed a chapter to a book titled 'Applications of Infrared to Biomedical Scieners', a prestigious venture by Nanyang Technological University, Singapore, published by Springer Nature. Singapore in 2017.She is committed to the cause of furtheringquality research and is a recogntred supervisor for guiding M.S / Ph.D scholars with Anna University, Chennai.
M.A.M SCHOOL OF ENGINEERING

Siruganur, Trichy-621105
Department of Mechatronics Engineering

## Webinar on "Principles of Robotics and its Applications" (18 ${ }^{\text {th }}$ MAY, 2020)

Department of Mechantronics Engineering had the privilege of having webinat with Dr.Sheeja.V.Francis, M.S., Ph.D Prof \& Head Department of Bio Medical Engineering, Jerusalem College of Engineering, Chennai, on the topic of "Principles of Robotics and its Applications".

The invitation for this program by the designer team of MAMSE and distributed through face book. The registration form for this program has created by Google form and published in our college website on $16^{\text {th }}$ May 2020.

The link for the registration:
www.mamse.in

Totally 316 participant has register for this webinar. The session is started sharply by 3.00 pm from welcome address and introduction given by Dr.Punitha, Professor, Mechatronics Engineering Department. After that the resource person starts the lecture with the basics of Robot configurations, after that she extended lecture session with kinematics of Robotics, Industrial Robots Applications, Manipulator Controls, etc. The event ended with vote of thanks given by M.Suba pradha, Assistant Professor, Mechatronics Department Also the feedback for the participant was collected through Google form.

## Program Co-ordinator



MAM SCIIOOL ÖF ENGINEERING
DEPARTMENT OF MECHATRONICS
Participants Registration for the Webinar "Principles of Robotics and its Applications"










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| 311 | 2020051818132.12 PM G31T+5 30 | 5 Suchax | Jayaram college of engineering and technology, Trichy | Electrical and electroaicy engineering | Female |  | ¢¢TJT30N9 |
| 312 | 20200818 1 430s PM GMT+5.30 | L. Swethe | MAM $\times$ bool of engurering | Elatronics and conmunication enginecring | fenuic | Swehatrateace 38 Stmail con | \$620493481 |
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| Total No of Registration: 316 |  |  |  |  |  |  |  |

## MAM SCHOOL OF ENGINEERING

## EPARTMENT OF MECHATRONICS

## Feedback for the Webinar "Principles of Robotics and its Äpplications"




|  | 2020/05/18 3:53:20 PM GMT+5:30 | R. Prem kumar | M. A. M. School of Enginecring | Mcehanical | Faculty | Mechanical. Premogrmail.com | Excelient |
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| 51 | 20205818353.20 PM GMTV5.30 | RTomkum |  |  | Studen | shanku1 3079968mail com | Excellent |
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| 74. | 2020:05/18 4:00:14 PM GMTr $+5: 30$ | Jenith kumar. B | MA.M School of Engineering: : | Mechatronics | Student | jenithbala07@gmail.com | Excellent |
| 75. | 20200548.4:15:21 PM, | SAM CHARLES DEVAPRASAD | Mahendrainstitute of engineering and.technology.---.- | Mechanical enginecring. | Faculty .-. | samchaclesjomiel asia $\ldots \ldots . \ldots$ | Excellent. |
| 76 | 2020:05/184:22:40. PM.CMI 5 5:30 | NISHASHREE TR. | JERUSALEM COLLEGEOE ENGINEERING. | BIOMEDICAL ENGINEEBING | Student | nishashreetresmail com | Excellent. |
| $\pi$ | $2020051185: 77.46 \mathrm{PMGMT}+5.30^{-}$ | K.pitabhazaran. | Jenusatem contege orengineering .-. | Bomedicat engtnecring | Stuax | KK792933才8matcom | cosd |


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# DEPARTMENT OF MECHATRONICS ENGINEERING 

## Webinar <br> On

## "Role of Microcontrollers in Mechatronics Systems"

### 09.06.2020

# Webinar On <br> Role of Microcontrollers in Mechatronics Systems 



Date: 09-06-2020 TIME: II. 30 am.

# Dr. S. Mohanalakshmi, Prof \& Head/ECE, 

Rohini college of Engineering and Technology kanyakumari.

E - Certificate will be provided forthe participants

## For Registration Visit www.mamse.in

## Resource Person Profile



Dr. S. Mohanalakshmi is an educator and researcher with 23 years of experience in teaching and research. She received her Ph.D under the faculty of Information and Communication Engineering from CEG, Anna University, Chennai and Masters in Applied Electronics from Sathyabama University, Chennai. Her area of interest includes Signal and Image processing, IoT, Embedded Systems and Biomedical Engineering. She holds 25 publications to her credit which includes Journals and Conference Proceedings. She is part of the Reviewer Board of the Journal Biomedical Engineering/Biomedizinische Technik, Germany and life member of IETE. She currently serves as Professor and Head of the Dept./ ECE in Rohini college of Engineering and Technology, Kanyakumari, TN.
M.A.M SCHOOL OF ENGINEERING

Siruganur, Trichy-621105
Department of Mechatronics Engineering

## Webinar on "Role of Microcontrollers in Mechatronics Systems" ( $9^{\text {th }}$ June, 2020)

Department of Mechatronics Engineering had the privilege of having webinar with Dr.S.Mohanalakshmi, Professor and Head/ECE, Rohini college of Engineering and Technology, Kanyakumari on the topic of "Role of Microcontrollers in mechatronics systems".

The invitation for this program by the designer team of MAMSE and distributed through face book. The registration form for this program has created by Google form and published in our college website on $8^{\text {th }}$ June 2020.

The link for the registration:
https://docs.google.com/forms/d/e/1FAlpQLScNHcW7mn1DeolZ1V 4mJLJLVI sYOhnlg71HaLseLd4HmRhKA/viewform

Totally 48 participant has register for this webinar. The session is started sharply by 11.30 Am from welcome address and introduction given by P.Sudha, Assistant Professor, Mechatronics Engineering Department. After that the resource person starts the lecture with the introduction of microcontrollers. Later she given the lecture with demonstration of how microcontrollers are featured with Mechatronics design, challenging controls of the microcontrollers in various parameter etc. The event ended with vote of thanks given by M.Suba pradha, Assistant Professor, Mechatronics Department. Also the feedback for the participant was collected through Google form.





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[^2]:    ${ }^{1}$ The set $\mathcal{F}$ may be specified by equations of the form (1.1) and/or (1.2).
    ${ }^{2}$ Alternatively, the term global minimiser can be used to denote a point at which the function $f$ attains its global minimum.

[^3]:    ${ }^{3} \mathrm{~A}$ compact set is a bounded and closed set.

[^4]:    ${ }^{4}$ We denote with $\nabla f$ the gradient of the function $f$, i.e. $\nabla f=\left[\frac{\partial f}{\partial x^{1}}, \cdots, \frac{\partial f}{\partial x^{n}}\right]{ }^{\prime}$. Note that $\nabla f$ is a column vector.

[^5]:    ${ }^{5}\|x\|_{E}$ denotes the Euclidean norm of the the vector $x$, i.e. $\|x\|_{E}=\sqrt{x^{\prime} x}$.

[^6]:    ${ }^{7}$ We denote with $\|v\|_{E}$ the Euclidean norm of the vector $v$, i.e. $\|v\|_{E}=\sqrt{v^{\prime} v}$.

[^7]:    ${ }^{8}$ This is equivalent to say that $\frac{\partial F}{\partial x}(x)$ is Lipschitz continuous in $\mathcal{D}$.

[^8]:    ${ }^{9}$ If $f$ is quadratic then $\rho_{k}=1$ for all $k$.

[^9]:    ${ }^{1}$ The condition $h(x) \leq 0$ has to be understood element-wise, i.e. $h_{i}(x) \leq 0$ for all $i$.

[^10]:    ${ }^{2}$ We denote with $\nabla_{x} f$ the vector of the partial derivatives of $f$ with respect to $x$.

[^11]:    ${ }^{3}$ Note that $m$ is the number of the equality constraints, and that, to avoid trivial cases, $m<n$.

[^12]:    ${ }^{4}$ This is the case if $\operatorname{rank} A=m$.

[^13]:    ${ }^{5}$ Because of the discontinuity of $F$, the limit has to be considered with proper care.
    ${ }^{6}$ The set $\mathcal{X}^{\sigma}$ is sometimes called the relaxed admissible set.

[^14]:    ${ }^{7}$ To be precise we should write $L_{a}\left(x, \lambda^{\star}, \epsilon\right)$, however we omit the argument $\epsilon$.

[^15]:    ${ }^{8}$ As in previous sections we omit the argument $\epsilon$.

[^16]:    ${ }^{9}$ The set $\tilde{\mathcal{X}}$ is defined as in equation (3.21).

[^17]:    Austenite

    - High temperature phase
    - Cubic crystal structure

